

Philadelphia University

Electrical Engineering Department

Small Electrical Machines

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Small Electrical Machines

Dr. M.T. Lazim

Introduction

Electrical Machine is defined as an electromagnetic device converting:

- (a) Mechanical energy to electrical energy (GENERATOR).
- (b) Electrical energy to mechanical energy (MOTOR).
- (c) Electrical energy to electrical energy at different voltage, current, frequency.. etc. (TRANSFORMERS, energy converters).

Classifications and Types of Electrical Machines:

Due to the importance of electrical machines in industrial and domestic use, electrical machines can be classified in various ways as follows:

1. According to the type of motion:

- (a) Rotary machines
- (b) Static machines
- (c) Linear motion machines

2. According to the function

- (a) Generator
- (b) Motor
- (c) Transformer
- (d) Energy Converters
 - (i) AC-DC converters (Rectifiers)
 - (ii) DC-AC converters (Inverters)
 - (iii) DC-DC converters (choppers)
 - (iv) AC-AC Converters (AC-choppers & Cycloconverters)

3. According to the supply :

- (a) DC Machines
- (b) AC Machines
 - Single-phase
 - poly-phase

4. According to the principle of action :

- (a) Synchronous Machines
- (b) Asynchronous Machines
 - Induction machine
 - Commutator machine

5. According to the use

- (a) Machine for general use
- (b) Machine for special use { Servomotors, synchros, tachogenerators, Selsyns, resolvers --- etc }.

Size of Electrical Machines:

Electrical Machine sizes could be from few milliwatts up to 2000 MW. In general, they are classified according to the size as follows:

1. Micromachines
2. Subfractional horse power machines
3. Fractional horse power machines.
4. Small machines
5. Medium machines
6. Large machines.

Our Course will deal with micromachines, subfractional and fractional horse power machines as well as the small machines.

Small Electrical Machines
Micromachinery

DC Machines

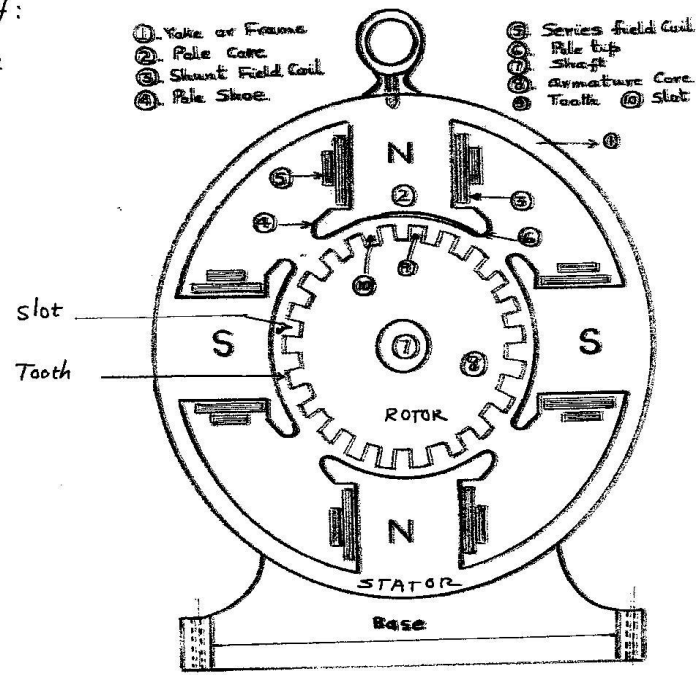
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A DC machine essentially has the following Parts:

1. Stator or field system.

2. Rotor consists of:

- i) Armature Core and windings
- ii) Commutator
- iii) Brush gear



Cross Section of a 4-Pole d.c machine

STATOR

Field system consists of:

- a) Yoke:- made of either cast iron or cast steel to carry the poles and ensure a low reluctance path for the magnetic field.
- b) The poles: consist of a pole core and pole shoe. The pole core is made either of cast iron or sheet steel. The pole shoe made of sheet steel.

DC/1

C) The Field Winding : Coils are wound around the pole core and carried by the pole shoe. These coils are supplied from a d.c. Source or from the machine itself by direct current.

The ampere turn (AT) produced by this current create the main magnetic field.

- The yoke, the poles and the field windings are usually called STATOR.

Armature consists of :-

a) Slots and teeth : In the slots the armature windings are placed.

b) Armature windings :- They are the machine windings or conductors and consist of a large number of coils.

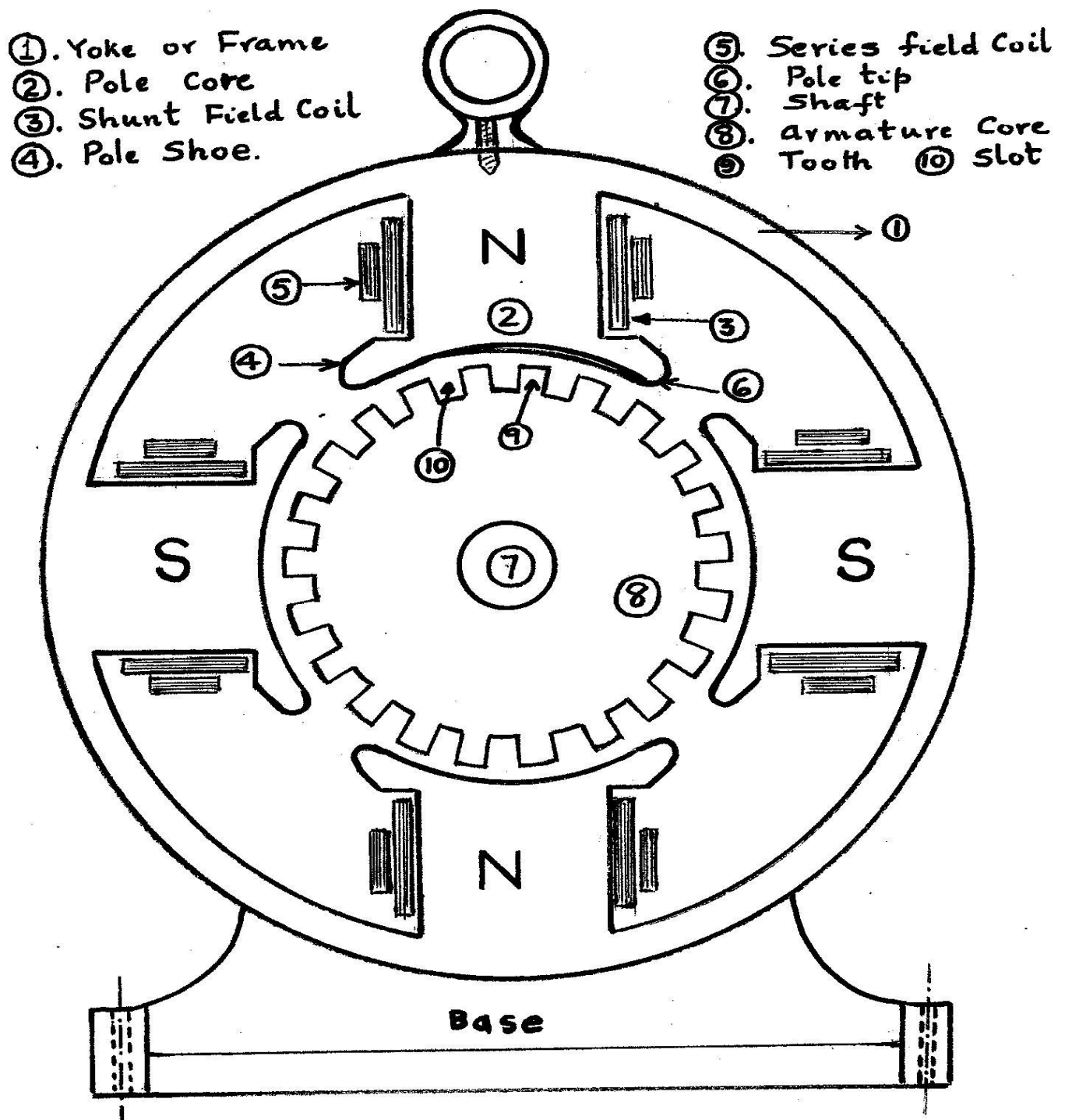
c) Armature core : is made of from sheet steel laminations, (0.4 - 0.5) mm thick.

The armature core is worked at a flux density about 1.2 to 1.3 Wb/m² (Tesla).

Commutator : The commutator is made of hard drawn copper wedge-shaped segments insulated from each other by thin mica layers. The segments are tapered and held between V-shaped rings to prevent their flying out due to centrifugal force.

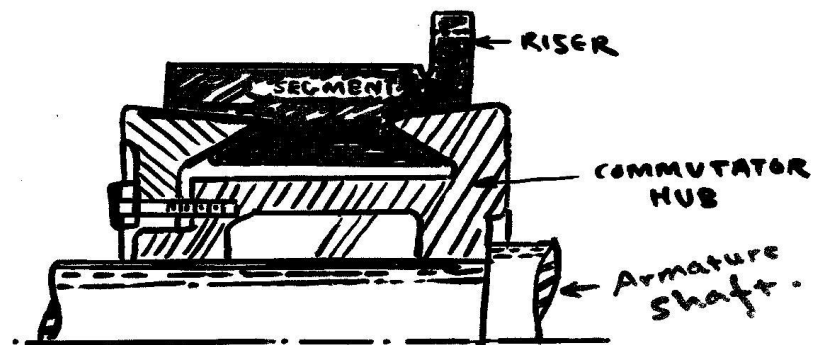
The commutator + the armature called ROTOR.

Dc/2



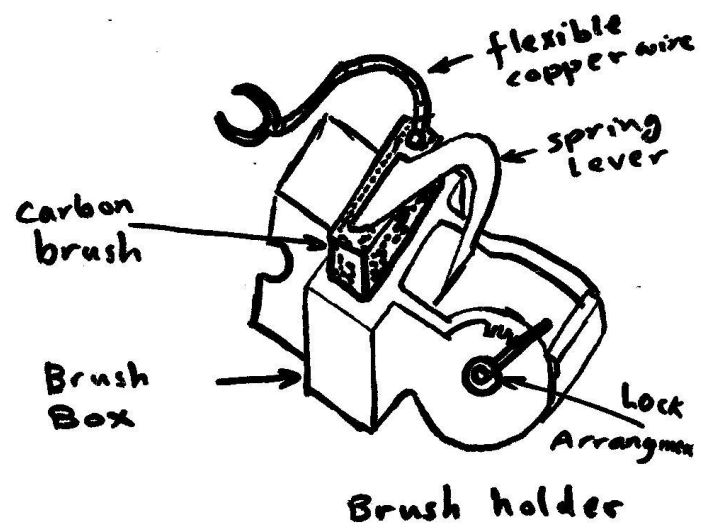
Cross Section of a 4-Pole d.c. machine

DC/3



Brush gear

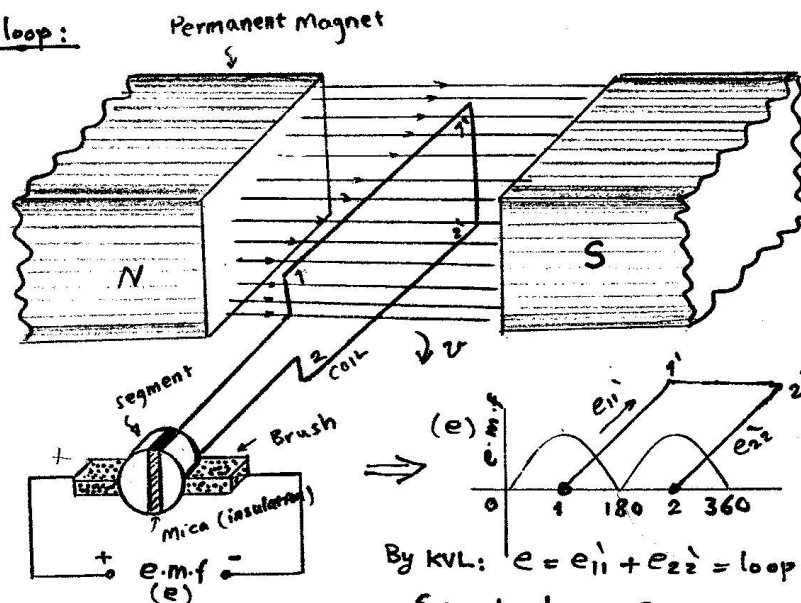
The brushes are held in position by box-type holders attached to the stator frame. The brushes are made of carbon. The current collected by the brushes is delivered to the external circuit by flexible copper wires.



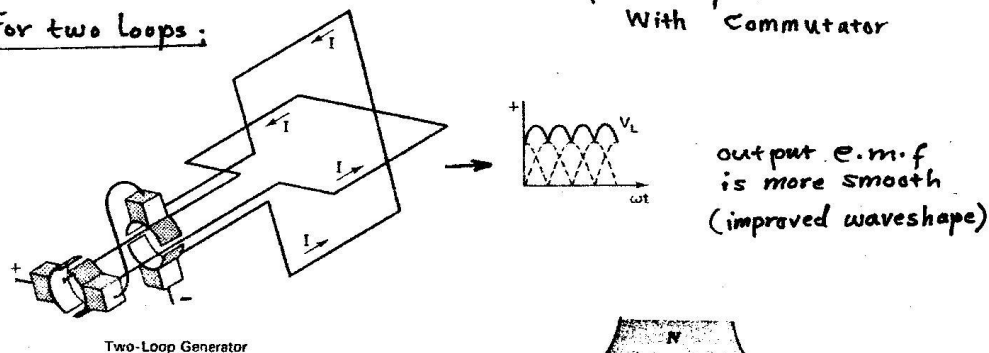
DC/4

Generator principle :

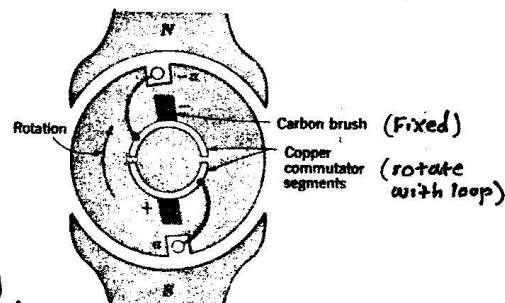
Consider a wire loop $11', 22'$ rotating at a uniform speed in a magnetic field Φ . According to Farady's Law: $e \propto \frac{d\Phi}{dt}$. conductors $11'$ and $22'$ are the active parts of the loop, the remaining parts are "end-connections".

For single loop:

- Simple Loop Generator
With Commutator

For two loops:

- slip rings and brushes may be used to make electrical contact with a rotating loop. slip rings rotate with loop, while brushes are stationary to give sliding contacts.
- Commutator is a conducting ring split into segments, each segment is electrically connected to the loop.



Elementary dc machine with commutator

DC/5

Small Electrical
Machines

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DC MACHINES

ARMATURE WINDINGS

Armature windings are mostly accommodated on the rotor of the machine so that the commutator and brush combination can rectify the voltage.

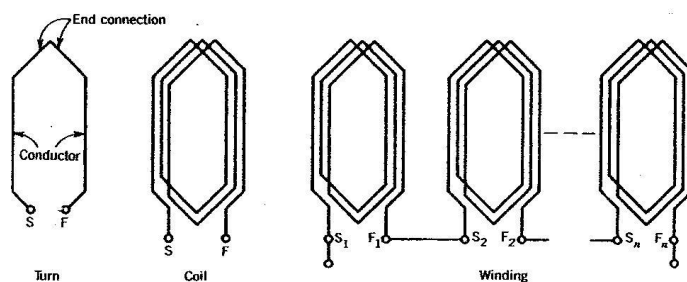
There are various ways to construct an armature winding. But before discussing these ways, some basic terms related to it are defined.

Turn :- Each two conductors connected together to form one turn.

Coil :- Coil is formed by connecting several turns in series.

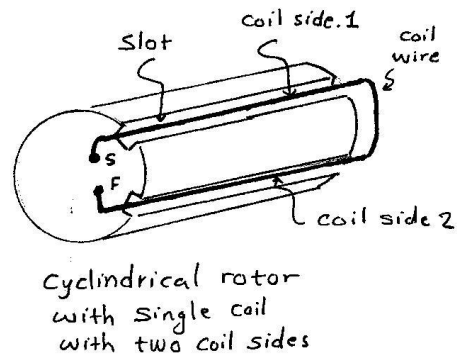
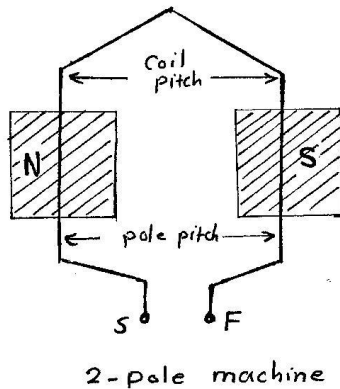
Winding :- winding is formed by connecting several coils in series.

The beginning of the turn, or coil is identified by the symbol S and the end by symbol F.



DC/6

The two coil sides are placed in two slots on the rotor surface. The distance between the two sides of a coil is called Pole pitch or coil pitch.



Total number of conductors

To calculate the number of the total conductors in a d.c. machine :

Let

C = number of coils in a machine

$\therefore 2C$ = number of coil sides

If N = number of turns per coil

$\therefore NC$ = total number of turns

If Z = total number of conductors

$\therefore Z = 2NC$

DC/7

Lap winding & Wave winding

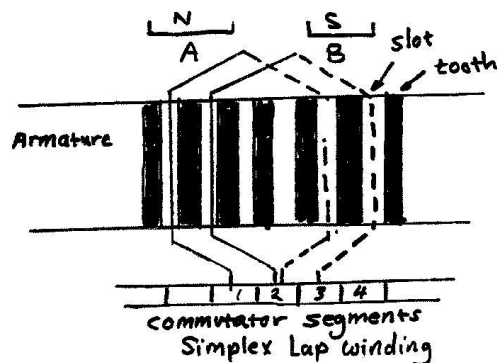
There two types of d.c. machine winding of drum type :

1. Lap winding
2. Wave winding.

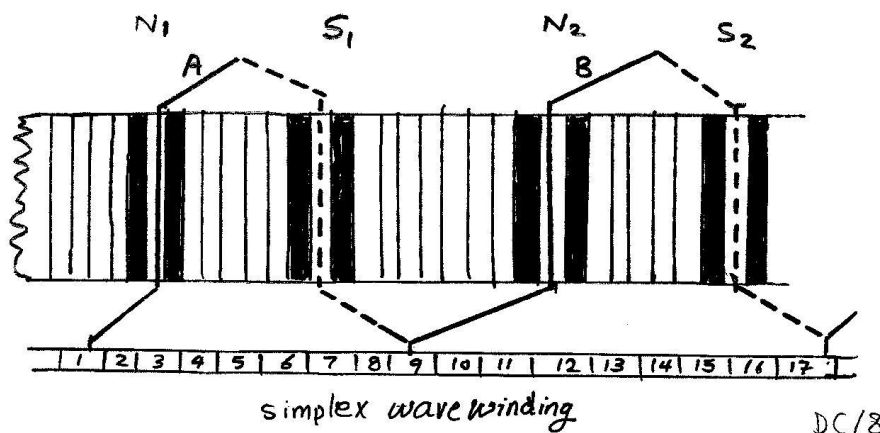
Both windings are made from simple coils or winding element.

The two windings are distinguished from each other by the way in which the leads of the coils are connected to the commutator segments.

- In the Lap winding coil B lap coil A, i.e coil A under a pair of poles (N,s) is connected in series with another coil B under the same pair of pole.



- In the wave winding coil A is placed such that it has one side under the north pole N_1 , while the other side is under the south pole S_1 . Coil B has to be connected to coil A has its sides are under the adjacent pair of poles (N_2, S_2).



DC/8

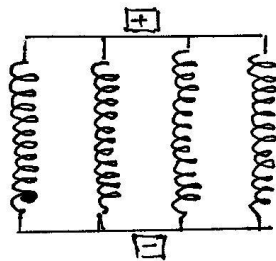
Number of parallel paths (α)

- In a lap winding, the number of parallel paths (α) is always equal to the number of poles (p).

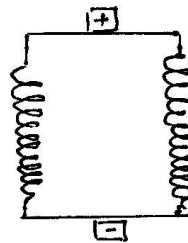
$$\therefore \alpha = p \quad \text{for Lap Winding.}$$

- In a wave winding, the number of parallel paths (α) is always = 2.

$$\alpha = 2 \quad \text{for wave winding}$$



4-pole machine
Lap winding
 $\alpha = 4$



4-pole machine
Wave winding
 $\alpha = 2$

Armature Voltage (E.M.F Equation)

The induced voltage in the armature winding of a d.c. machine can be derived directly from Farady's law of electromagnetic induction as follows:

Let:

Z = total number of conductors on the armature

α = number of parallel paths.

p = number of field poles

Φ = flux per one pole.

n = speed of rotation of the armature in revolution per minute (r.p.m)

ω = speed of rotation in rad/sec = $\frac{2\pi n}{60}$

DC/9

Then :

$$\begin{aligned}\text{Flux cut by one conductor in one revolution} &= \Phi p \\ \text{Flux cut by one conductor in } n \text{ revolutions} &= \Phi p n \\ \text{Flux cut by one conductor per one second} &= \frac{\Phi p n}{60} \\ \text{number of conductors in series} &= \frac{Z}{a}\end{aligned}$$

$$\text{Flux cut per one second by } \frac{Z}{a} \text{ conductors} = \frac{\Phi p n}{60} \left(\frac{Z}{a} \right)$$

Hence, the e.m.f induced in the armature winding :

$$E = \left(\frac{p}{60} \right) \left(\frac{Z}{a} \right) \cdot n \Phi \quad \text{Volts}$$

$$\text{or } \boxed{E = K_e n \Phi} \quad \text{Volts}$$

where K_e = constant, called the machine constant.

Example: An 8-pole, lap-connected armature has 960 conductors. The flux per pole is 40 mWb, and the armature rotates at 400 r.p.m. Calculate the e.m.f generated on open circuit.

Solution:

For lap wound armature $a = p = 8$ parallel paths

$Z = 960$ conductor

$n = 400$ r.p.m

$$\therefore E = \frac{p}{60} \times \frac{Z}{a} \cdot n \cdot \Phi$$

$$= \frac{8}{60} \times \frac{960}{8} \times 400 \times 40 \times 10^{-3} = \underline{\underline{256 \text{ V}}}$$

DC/19

Torque Equation for a DC machine

The torque developed in a d.c. machine can be calculated from the principle of reversibility of the machine which state that :

In a DC machine, the electrical power input or output always equal to the mechanical power input or output, i.e.:

$$P_E = P_M$$

$$\text{or } EI_a = T_e \omega$$

where E : is the generated e.m.f in the armature .

I_a : is the armature current .

T_e : is the developed or electromagnetic torque.

ω : is the speed of the armature in rad/sec.

$$\text{Since } \omega = \frac{2\pi n}{60} \quad (n \text{ is the speed in r.p.m})$$

$$\& \quad E = \frac{P}{60} \times \frac{Z}{a} \cdot n \Phi$$

Thus

$$\frac{P}{60} \times \frac{Z}{a} n \Phi I_a = T_e \left(\frac{2\pi n}{60} \right)$$

$$\boxed{T_e = \frac{Z P}{2\pi a} \Phi I_a}$$

$$\text{or } T_e = K_T \Phi I_a$$

where

$$K_T = \frac{Z P}{2\pi a}$$

DC/11

Example 1: A six-pole DC machine has a flux per pole of 15 mWb. The armature has 536 conductors connected in lap winding. The DC machine runs at 1050 rpm and it delivers a rated armature current of 22.5 A to a load connected to its terminals. Calculate:

- (a) Machine constant K_e
- (b) Generated voltage E
- (c) Conductor current I_c
- (d) Electromagnetic torque T_e
- (e) Power P_d developed by the armature.

Solution: For lap winding $a = p$

$$(a) \quad K_e = \frac{P}{60} \times \frac{Z}{a} = \frac{6}{60} \times \frac{536}{6} = 8.93$$

$$(b) \quad E = K_e \Phi n = 8.93 \times 15 \times 10^{-3} \times 1050 = 140.7 \text{ V}$$

$$(c) \quad \text{Conductor current } I_c :$$

For lap winding there are $a = 6$ parallel paths, therefore the conductor current is

$$I_c = I_{\text{coil}} = \frac{I_a}{a} = \frac{22.5}{6} = 3.75 \text{ A}.$$

$$(d) \quad T_d = \frac{Z P}{2\pi a} \Phi_a I_a$$

$$= \frac{536 \times 6}{2\pi \times 6} \times (15 \times 10^{-3}) \times 22.5 = 28.79 \text{ N.m}$$

$$(e) \quad \text{power developed by the armature}$$

$$P_d = P_E = E I_a = 140.7 \times 22.5 = \underline{\underline{3165.75 \text{ W}}}.$$

$$\text{or } P_d = P_M = T_e \cdot \omega = 28.79 \times \frac{2\pi n}{60}$$

$$= 28.79 \times \frac{2\pi \times 1050}{60} = \underline{\underline{3165.75 \text{ W}}}$$

Same answer.

H.W2: Repeat the above example if the machine is reconnected in wave winding.

DC/12

Example 2: A 4-pole small electrical DC machine has an armature of a radius 6 cm and effective length of 10 cm. The poles cover 75% of the armature periphery. The armature winding consists of 22 coils, each coil have six turns. The coils are accommodated in 22 slots. The average flux density under each pole is 0.35 Tesla. If the armature is Lap-wound, determine:

- The induced armature voltage when the armature rotates at 3000 rpm.
- The current in the coil and the electromagnetic torque when the armature current is 12 A.
- The power developed by the armature

Solution:

$$(a) \quad E = K_e n \Phi$$

$$K_e = \frac{P}{60} \times \frac{Z}{a}$$

For lap-wound machine $a = p = 4$

$$\begin{aligned} Z &= 2 \times \text{No. of coils (c)} \times \text{No. of turns (N)} = 2NC \\ &= 2 \times 22 \times 6 = 264 \text{ conductors.} \end{aligned}$$

$$\therefore K_e = \frac{4}{60} \times \frac{264}{4} = 4.4$$

$$\text{Pole area } A_p = \frac{\pi d L}{p} \times 0.75$$

where d = armature diameter = 2x6 cm

L = armature length = 10 cm

$$\therefore A_p = \frac{2\pi \times 6 \times 10^{-2} \times 10 \times 10^{-2}}{4} \times 0.75 = 7.065 \times 10^{-3} \text{ m}^2$$

$$\Phi_p = A_p \cdot B = 7.065 \times 10^{-3} \times 0.35 = 2.47 \text{ mWb}$$

$$\therefore E = K_e \Phi_p n = 4.4 \times 2.47 \times 3000 \times 10^{-3} = 32.64 \text{ V}$$

DC/13

$$b) \quad I_{coil} = \frac{I_a}{a} = \frac{12}{4} = 3A .$$

$$\begin{aligned} T_e &= K_T \Phi_p I_a = \frac{Z P}{2\pi a} \Phi_p I_a \\ &= \frac{264 \times 4}{2\pi \times 4} \times 2.47 \times 10^{-3} \times 12 \\ &= 1.246 \text{ N.m} \end{aligned}$$

(c) Power developed:

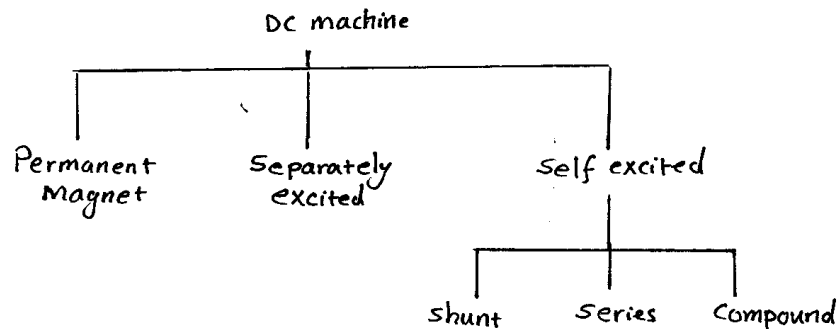
$$P_d = I_a E = 12 \times 32.64 = 391.68 \text{ W} .$$

$$\text{or } P_d = T_e \cdot \omega = 1.246 \times \frac{2\pi \times 3000}{60} = \underline{\underline{391.68 \text{ W}}} .$$

DC/14

Small Electrical Machines	Dr.M.T.Lazim
Types of DC Machines	

According to the type of magnetic field used in a d.c. machine, it may be classified as follows:



- In permanent magnet dc machine, the stator wound poles are replaced by permanent magnet. The rotor has a conventional dc armature, with commutator segments and brushes.

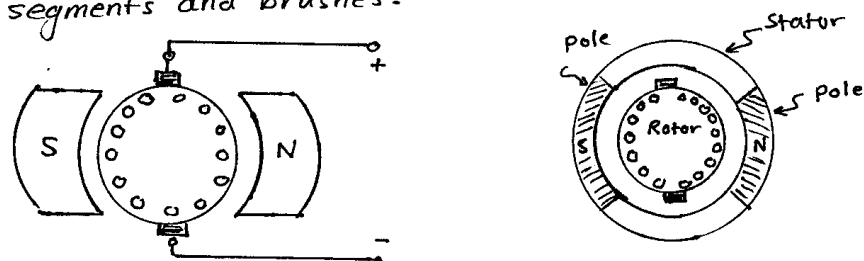
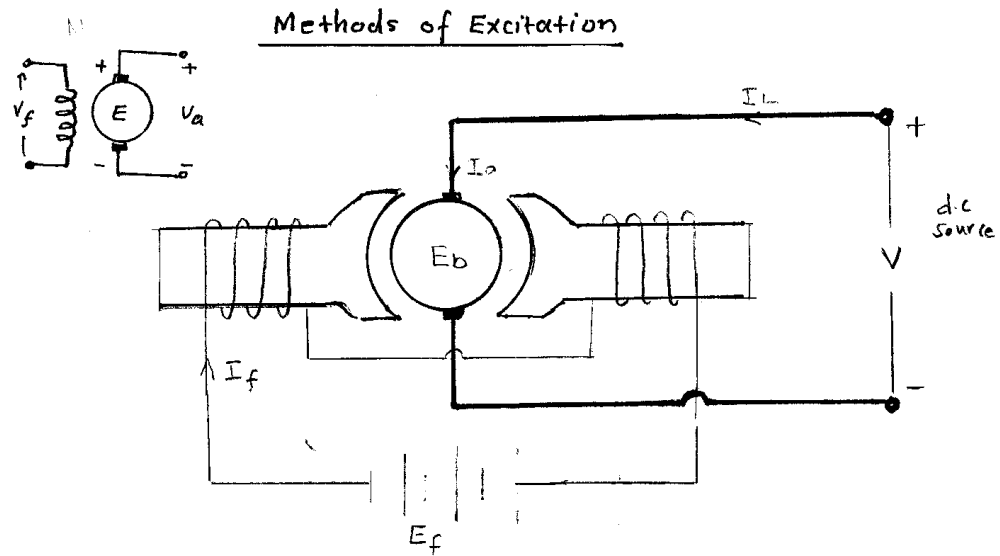


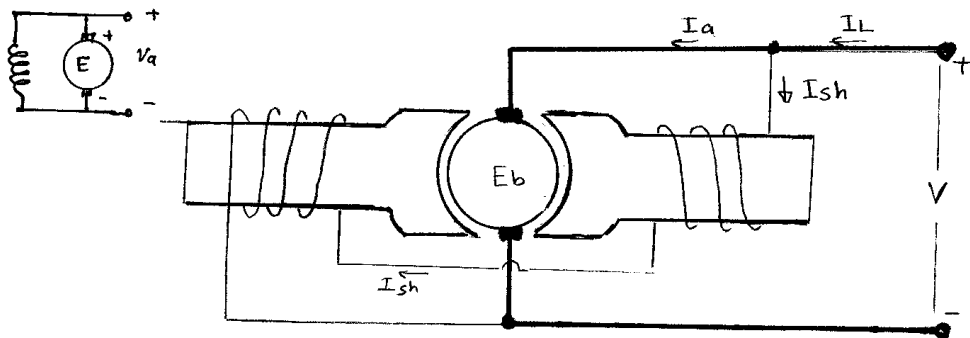
Fig.A Permenen Magnet dc. machine

- In electric magnet d.c. machine, the field circuit and armature circuit can be interconnected various ways to provide several types of d.c.generator or motor. These are 1. Separately excited 2. Shunt excited and Series excited as shown in Fig.B.

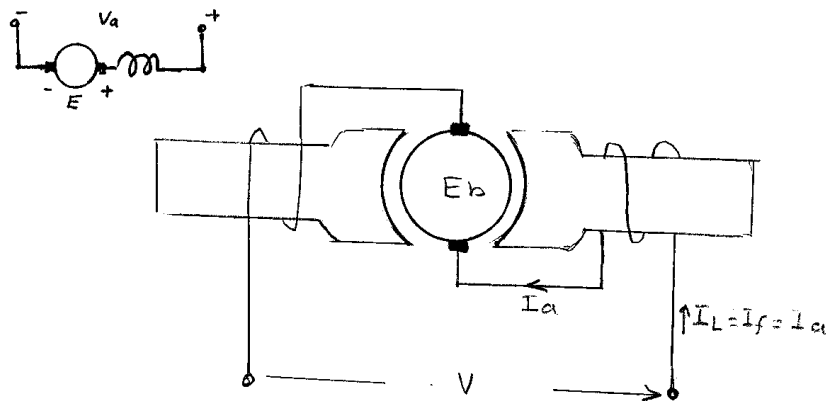
DC/15



1. Separately excited machine



2. Shunt excited machine



3. Series excited machine

Fig. B

DC/16

Types of DC Motors

According to the field connection arrangements, there are three main types of DC motors:

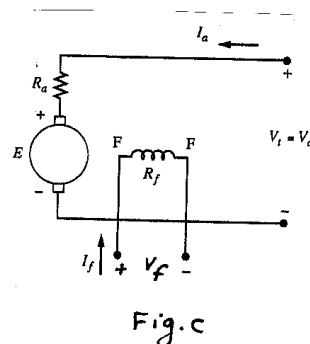
1. Separately Excited Motor :

Fig. C shows a typical separately excited motor.

In this type of motor, the field winding FF is connected to an external (separate) source V_f .

The equations of this motor are :

$$\begin{aligned} I_f &= \frac{V_f}{R_f} \\ V_t &= E + I_a R_a \\ I_a &= I_L \end{aligned} \quad (1)$$



where :

I_f = field circuit current.

R_f = field circuit resistance.

V_t = motor terminal (supply) voltage.

E = generated (back) e.m.f in the armature.

I_a = armature current.

I_L = motor current.

R_a = armature resistance.

Note : $E = k_e n \Phi = \frac{P}{60} \times \frac{Z}{a} n \Phi$

2. Shunt Motor

In shunt motor, the field winding is connected in parallel with the armature circuit as shown in Fig.D.

- The equations governing the dc shunt motor are:

$$I_L = I_a + I_f$$

$$I_f = \frac{V_t}{R_f + R_{rh}}$$

$$V_t = E + I_a R_a$$

where : R_{rh} is a rheostat (external) resistance connected in series with the field winding.

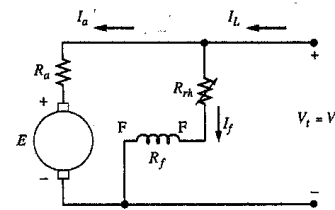


Fig.D

3. Series Motor

In series motor, the field winding is connected in series with the armature.

- The equations governing the series motor are:

$$V_t = E + (R_a + R_s) I_a$$

$$I_L = I_a$$

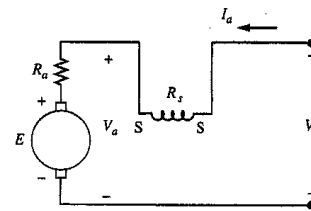


Fig.E

where: R_s : is the series winding resistance.

Note: shunt and series machines are self excited type.

Power Flow and Efficiency of Small Electrical Machine

The power flow in a small d.c. machine is shown in Fig. F. With the machine operating as a motor, the input power is electrical taken from the supply.

- Part of electrical input power is lost in R_a and part lost in the field resistance R_f .
- The remaining power $E I_a$ is converted into mechanical power.
- Part of the mechanical power is lost as rotational losses required to rotate the machine against friction and windage.
- The remaining power is appearing as mechanical or shaft power.
- The efficiency of the machine is given by:

$$\eta = \frac{P_{out}}{P_{in}} = \frac{\text{Output power}}{\text{Input power}} = \frac{P_{out}}{P_{out} + P_{loss}}$$

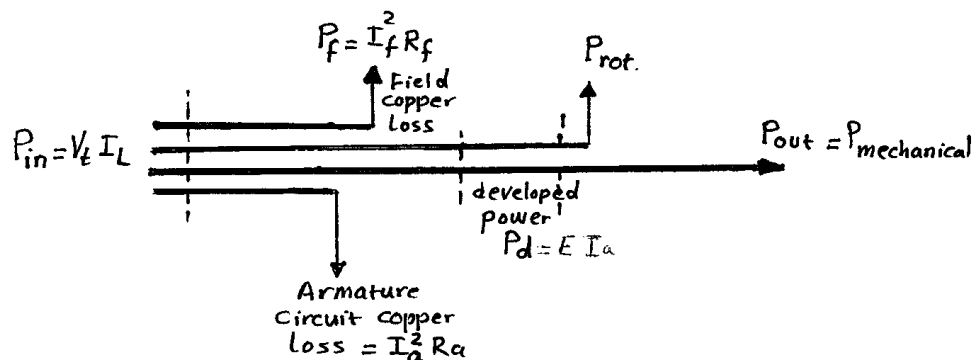


Fig. F

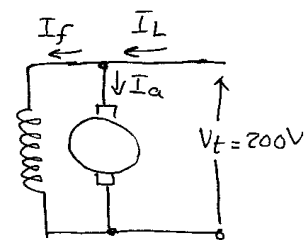
- Example 2: A small d.c. shunt motor is rated for 200V, 1350 r.p.m., 1000 W, $I_L = 6.5$ A and $I_f = 0.2$ A. It is known that $R_a = 0.35 \Omega$ and $P_{rot} = 51.9$ W at rated speed.
- (a) Determine the rated conditions developed torque, the generated e.m.f. (E) and the efficiency.
- (b) If torque on the motor shaft is reduced such that the motor draws a current of $I_L = 2.5$ A, calculate the value of the developed torque and the speed.

Solution

$$T = \frac{P_d}{\omega} = \frac{P_{out} + P_{rot}}{\omega}$$

$$= \frac{1000 + 51.9}{\frac{2\pi \times 1350}{60}} = \frac{1051.9}{141.3}$$

$$= 7.44 \text{ N.m}$$



By KCL:

$$I_L = I_a + I_f$$

$$\text{or } I_a = I_L - I_f = 6.5 - 0.2 = 6.3 \text{ A.}$$

$$E = V_t - I_a R_a = 200 - 6.3 \times 0.35 = 197.795 \text{ V.}$$

$$P_{in} = I_L V_t = 6.5 \times 200 = 1300 \text{ W.}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{1000}{1300} \times \frac{100}{100} = 76.9\%.$$

$$(b) \quad I_f = \text{constant} = 0.2 \text{ A.}$$

$$I_{a2} = I_{L2} - 0.2 = 2.5 - 0.2 = 2.3 \text{ A.}$$

$$\frac{T_{d2}}{T_{d1}} = \frac{K_T \Phi I_{a2}}{K_T \Phi I_{a1}} = \frac{I_{a2}}{I_{a1}}$$

$$\therefore T_{d2} = \left(\frac{I_{a2}}{I_{a1}} \right) T_{d1} = \left(\frac{2.3}{6.3} \right) (7.44) = 0.39 \times 7.44 = 2.95 \text{ N.m}$$

$$E_2 = V_t - I_{a2} R_a = 200 - (2.3)(0.35) = 199.2 \text{ V.}$$

$$\frac{E_2}{E_1} = \frac{K_e \Phi n_2}{K_e \Phi n_1} \quad \text{or } n_2 = \frac{E_2}{E_1} n_1 = \frac{199.2}{197.79} \times 1350 = 1360 \text{ r.p.m.}$$

DC / 21

Example: A 12 kW, 100 V, 1000 r.p.m dc shunt motor has armature resistance $R_a = 0.1 \Omega$, shunt field resistance $R_f = 80 \Omega$, is connected to 100 V d.c. supply. At no load condition, the motor runs at 1000 r.p.m and the armature takes 6 A.

- (a) Find the rotational losses at 1000 rpm.
- (b) Find the speed, electromagnetic torque and efficiency of the motor when the machine is loaded by mechanical load such that it draws rated current in the armature.

Solution:

- (a) At no load $I_a = 6A$

$$E = V_t - I_a R_a \\ = 100 - 6 \times 0.1 = 99.4V$$

$$I_f = \frac{V}{R_f} = \frac{100}{80} = 1.25A$$

At no load the electromagnetic power developed is lost as rotational power:

$$P_{\text{rotational}} = E I_a = 99.4 \times 6 = 596.4W.$$

- (b) when the motor is loaded it take rated current at the armature:

$$I_a = I_{a_{\text{rated}}} = \frac{12000}{100} = \frac{kW}{V} = 120A.$$

$$E|_{NL} = 99.4V$$

$$E|_{FL} = V - I_a R_a = 100 - 120 \times 0.1 = 88V.$$

DC /22

$$E = k_e n \Phi$$

$$\frac{E|_{FL}}{E|_{NL}} = \frac{k_e \Phi n_{FL}}{k_e \Phi n_{NL}} \quad (\Phi \text{ remains constant}).$$

$$= \frac{n_{FL}}{n_{NL}}$$

$$\therefore n_{FL} = \frac{E|_{FL}}{E|_{NL}} n_{NL} = \frac{88}{99.4} \times 1000 = 885.31 \text{ r.p.m.}$$

$$\omega_m = \frac{2\pi n}{60} = \frac{2\pi \times 885.3}{60} = 92.71 \text{ rad/sec.}$$

$$E I_a = P_d = T_e \omega_m$$

$$\therefore T_e = \frac{E I_a}{\omega_m} = \frac{88 \times 120}{92.71} = 113.9 \text{ N.m.}$$

$$P_{out} = P_d - P_{rotational} = E I_a - P_{rotational}$$

$$= 88 \times 120 - 596.4 = 9963.3 \text{ W.}$$

$$P_{in} = V I_m = V (I_a + I_f) = 100 (120 + 1.25) = 12125 \text{ W.}$$

$$\eta = \text{efficiency} = \frac{P_{out}}{P_{in}} = \frac{9963.3}{12125} = 0.8217$$

$$\% \eta = 0.8217 \times \frac{100}{100} = 82.17 \%$$

DC/23

D.C. Machines.

Example:

A 230V shunt motor has an armature resistance of 0.1Ω . This motor operates on the 230V mains at a speed of 1150 rpm and takes an armature current of 100A. An external resistance of 0.1Ω is now inserted in series with the armature, the field-rheostat setting remaining unchanged and the load being so adjusted that the armature current remains the same.

Compute the effect of this external resistance on the armature circuit input, armature e.m.f and speed.

Solution: Before the resistance is inserted:

$$\text{Armature Circuit Input} = V_t I_a = 230 \times 100 = 23 \text{ kW.}$$

$$E_{b1} = V_t - I_a R_a = 230 - 100 \times 0.1 = 220 \text{ V.}$$

After the resistance is inserted:

$$\text{Armature circuit input} = (230)(100) = 23 \text{ kW.}$$

$$E_{b2} = 230 - 100(0.1 + 1.0) = 120 \text{ V.}$$

Since the field current is constant, then ϕ is constant.

$$E_{b1} = k\phi n_1$$

$$E_{b2} = k\phi n_2$$

$$\therefore \frac{E_{b1}}{E_{b2}} = \frac{k\phi n_1}{k\phi n_2} \quad \text{or} \quad \frac{E_{b1}}{E_{b2}} = \frac{n_1}{n_2}$$

$$\text{or } n_2 = \frac{E_{b2}}{E_{b1}} n_1 = \frac{120}{220} (1150) = 627 \text{ rpm}$$

Ans.

Example: A 50 hp, 550 V, 750 rpm Series motor takes a current of 74 A at 550 V. Its armature resistance is 0.35Ω , and series field resistance ~~of~~ is 0.15Ω . When the load torque is double the rated value, the current is 110 A. Determine the speed and power output at the 100 percent torque overload. For this purpose, consider that doubling the shaft torque doubles the electromagnetic torque.

Solution: At rated load:

$$E_{b1} = 550 - 74(0.35 + 0.15) = 513 \text{ V}.$$

At the overload:

$$E_{b2} = 550 - 110(0.35 + 0.15) = 495 \text{ V}.$$

$$\text{At rated Load: } T_1 = K_T \phi_1 I_{a1}.$$

$$\text{At overload: } T_2 = K_T \phi_2 I_{a2}.$$

$$\therefore \frac{\phi_2}{\phi_1} = \frac{I_{a1}}{I_{a2}} \times \frac{T_1}{T_2} = \left(\frac{74}{110}\right) \times 2 = 1.346.$$

$$\text{but } E_{b1} = K_e \phi_1 n_1$$

$$E_{b2} = K_e \phi_2 n_2$$

$$\therefore \frac{E_{b2}}{E_{b1}} = \frac{\phi_2}{\phi_1} \times \frac{n_2}{n_1}$$

$$\text{or } n_2 = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2} n_1 = \frac{495}{513} \times \left(\frac{1}{1.346}\right) (750)$$

$$= \underline{537 \text{ rpm.}} \quad \text{Ans.}$$

(2) A shunt motor rotating at 1500 rpm is connected to 120 d.c. source. The line current is 51 A and the shunt field resistance is $120\ \Omega$. If the armature resistance is $0.1\ \Omega$, calculate the following:

- The armature current.
- The back (counter) e.m.f.
- The mechanical power developed by the motor.

Solution

- a) $I_L = 51\text{ A}$
 $R_a = 0.1\ \Omega$
 $V = 120\text{ V}$
 $R_f = 120\ \Omega$.

$$I_f = \frac{V}{R_f} = \frac{120}{120} = 1\text{ A}$$

$$I_a = I_L - I_f = 51 - 1 = 50\text{ A}$$

- b) $E_b = V - I_a R_a$
 $= 120 - 50 \times 0.1 = 120 - 5 = 115\text{ V}$.

- c) Power input to the motor

$$P_{in} = V I_L = 120 \times 51 = 6120\text{ W}$$

Power absorbed by the motor armature:

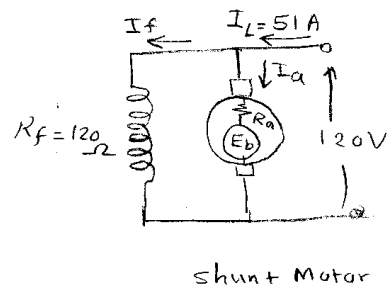
$$P_a = I_a V = 50 \times 120 = 6000\text{ W}$$

power loss in the armature ($R_a I_a^2$)

$$P_{cu} = (50)^2 \times 0.1 = 250\text{ W}$$

$$\therefore P_m = P_a - P_{cu} = 6000 - 250 = 5750\text{ W}$$

$$\text{or } P_m = \frac{5750}{746} = 7.7\text{ hp (horse power)}.$$



DC MOTORS CHARACTERISTICS

1. Separately excited d.c. motor

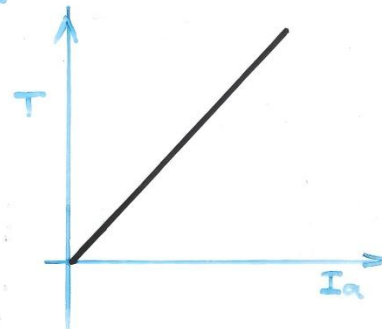
a. Torque - current ch/s :-

$$T = K_T \Phi I_a$$

$$K_T = \frac{1}{2\pi} \cdot \frac{2P}{a} \cdot \frac{Z}{\phi_0}$$

Φ = air gap flux

I_a = armature current



b. Speed - current ch/s:

Let:

V = Voltage applied to the armature terminal.

E_b = induced back e.m.f in the armature wdg.

ΣR_a = total resistance in the armature cct.

From motor equation:

$$V = E_b + I_a \Sigma R_a$$

$$\therefore I_a = \frac{V - E_b}{\Sigma R_a} = \frac{V - K_e n \Phi}{\Sigma R_a}$$

$$\text{where } K_e = \frac{2P}{2a} \cdot \frac{Z}{\phi_0}$$

$$I_a = \frac{V}{\Sigma R_a} - \frac{K_e n \Phi}{\Sigma R_a}$$

$$\text{or } n = \frac{V}{K_e \Phi} - \frac{\Sigma R_a}{K_e \Phi} I_a$$

At no load $I_a = 0$

$$\therefore n = \frac{V}{K_e \Phi} = n_0$$

At stand still

$$n = 0 \quad I_a = \frac{V}{\Sigma R_a}$$

In this case we call I_a as starting current

$$\therefore I_{\text{starting}} = \frac{V}{\Sigma R_a}$$

In general

$$n = n_0 - \frac{\Sigma R_a}{K_e \Phi} I_a$$

c. Torque - Speed ch/s
(Mechanical ch/s)

we have:

$$n = n_0 - \frac{\Sigma R_a}{K_e \Phi} I_a$$

$$T = K_T \Phi I_a$$

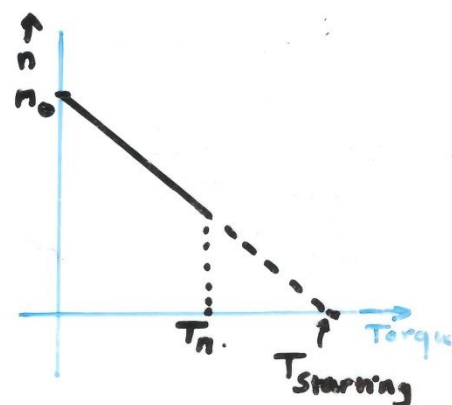
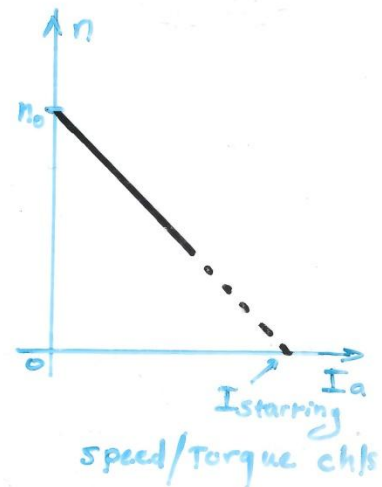
$$\therefore n = n_0 - \frac{\Sigma R_a}{K_e K_T \Phi^2} T$$

or

$$n = n_0 - m T$$

where

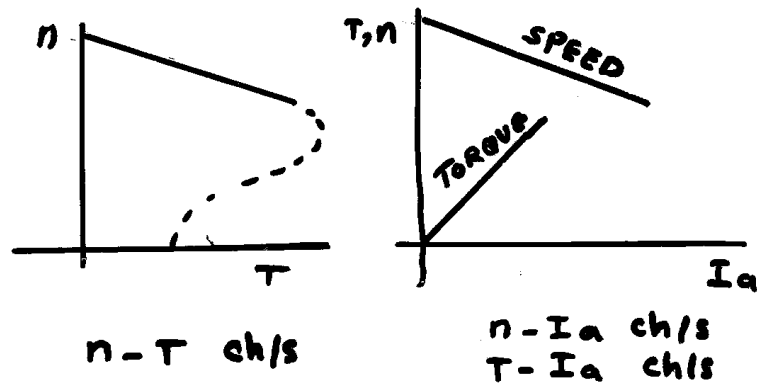
$$m = \frac{\Sigma R_a}{K_e K_T \Phi^2} = \text{slope}$$



$T_n = \text{nominal Torque}$

2. Shunt motor ch/s

shunt motor exhibit similar ch/s as that of separately excited motor.



3. Series Motor ch/s

$$T = K_T \Phi I_a$$

$$= K_T \Phi I$$

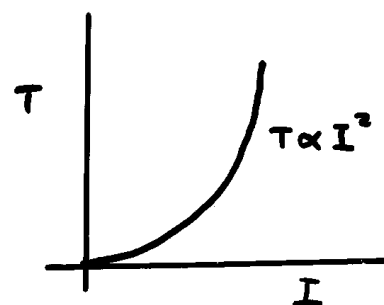
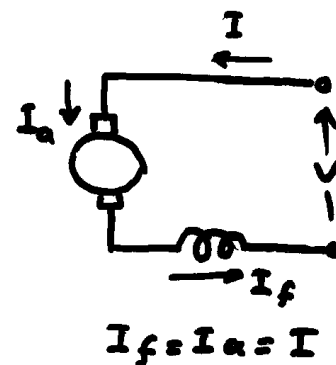
but $\Phi = K_f I_f$

$$\therefore T = K_T K_f I_f I$$

but $I_f = I$

$$\therefore T = K_T K_f I^2$$

The relation between
 T & I is parabolic
 ($y = x^2$).



Small Electrical Machines

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DC MOTORS

Speed Control of D.C. MotorsI. Conventional Methods

From the equation of the speed of DC motor:

$$n = \frac{V_t - \sum R_a I_a}{K_e \phi}$$

There are three basic ways of speed control:

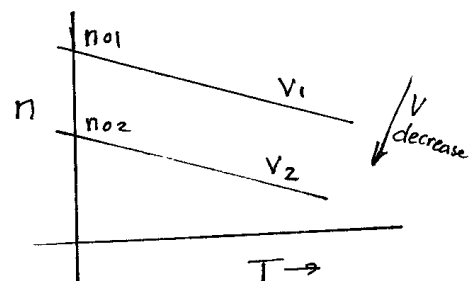
1. Variation of the applied voltage (V_t)
(Voltage Control)
2. Variation of the armature resistance ($\sum R_a$)
(Rheostatic Control)
3. Variation of the flux/pole (Φ)
(Field Control)

Thus For separately excited and shunt motors:

1. Variation of V_t :

$$n_1 = \frac{V_1 - \sum R_a I_{a1}}{K_e \phi}$$

$$n_2 = \frac{V_2 - \sum R_a I_{a2}}{K_e \phi}$$



Here $\sum R_a$ and ϕ are kept constant

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Micro machines	DC Machine Speed Control

2. Variation of ΣR_a [Rheostatic Control] Method.

If the external series resistance connected with the armature circuit of a d.c. motor is changed, while other quantities (V_t & ϕ) are kept unchanged, different values of speeds for d.c. motor may be achieved.

$$\Sigma R_a = R_{ext} + R_a$$

Where R_{ext} = external series resistance

R_a = actual armature resistance

Without R_{ext} :-

$$R_{ext} = 0$$

$$\therefore \Sigma R_a = R_a$$

$$\bullet \quad n_1 = \frac{V_t - I_{a1} R_a}{K_e \phi} \quad \dots \dots (1)$$

With R_{ext} :

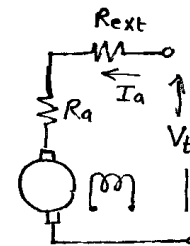
$$\Sigma R_a = R_{ext} + R_a$$

$$n_2 = \frac{V_t - I_{a2} (\overbrace{R_{ext} + R_a}^{\Sigma R_a})}{K_e \phi} \quad \dots (2)$$

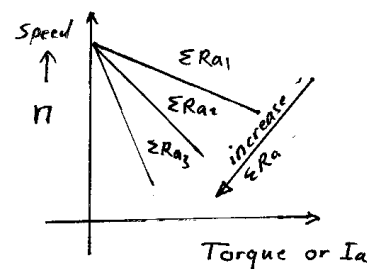
If ϕ is constant; Deviding eq.(1) by (2) :

$$\frac{n_2}{n_1} = \frac{V_t - I_{a2} \Sigma R_a}{V_t - I_{a1} R_a}$$

$$n_2 = \left[\frac{V_t - I_{a2} \Sigma R_a}{V_t - I_{a1} R_a} \right] n_1$$



dc. Separately excited motor



Example: A 120V, 1800 r.p.m, small shunt d.c. motor with an armature resistance of 0.5Ω is taking a current of 10A at rated load. Assuming the torque to be constant, calculate the value of external resistance placed in series with the armature to reduce the speed to 1000 r.p.m.

Solution :

As the torque remains constant, the armature current I_a should also remain constant, since $T \propto I_a$. Hence $I_{a1} = I_{a2}$.

$$n_1 = \frac{V_t - I_{a1} R_a}{K \phi}$$

$$n_2 = \frac{V_t - I_{a2} (R_{ext} + R_a)}{K \phi}$$

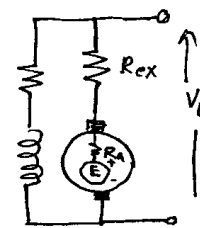
$$\therefore \frac{n_2}{n_1} = \frac{V_t - I_{a2} (R_{ext} + R_a)}{V_t - I_{a1} R_a}$$

$$\frac{1000}{1800} = \frac{120 - 10(R_{ext} + 0.5)}{120 - 10(0.5)}$$

$$1000 \times 115 = 1800 (120 - 10R_{ext} - 5)$$

$$1150 = 2070 - 180 R_{ext}$$

$$\therefore R_{ext} = \frac{920}{180} \Omega$$



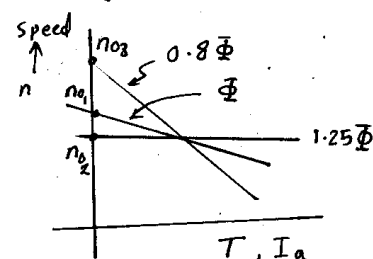
Shunt Motor.

3. Field Control (Flux Variation Method)

In this method V_t and R_a are kept constants, while Φ is varied.

$$n_1 = \frac{V_t - I_{a1} R_a}{K \Phi_1}$$

$$n_2 = \frac{V_t - I_{a2} R_a}{K \Phi_2}$$



$$\therefore \frac{n_2}{n_1} = \frac{V_t - I_{a2} \Sigma R_a}{V_t - I_{a1} \Sigma R_a} \cdot \frac{\Phi_1}{\Phi_2}$$

Note: Higher speeds can be obtained with field control.

Example 2 :

A 250V, Shunt motor runs at 800 r.p.m when taking an armature current of 20A. The armature resistance is 0.1Ω . Calculate the speed when a resistance is inserted in the field circuit as to reduce the flux to 80% of the nominal value. Assume the armature current is unchanged.

Solution

$$n_1 = \frac{V - I_{a1} \Sigma R_a}{k \Phi_1}$$

$$n_2 = \frac{V - I_{a2} \Sigma R_a}{k \Phi_2}$$

$$\therefore \frac{n_2}{n_1} = \frac{V - I_{a2} \Sigma R_a}{V - I_{a1} \Sigma R_a} \times \frac{\Phi_1}{\Phi_2}$$

Since $I_{a2} = I_{a1}$

$$\text{Hence } \frac{n_2}{n_1} = \frac{\Phi_1}{\Phi_2} \quad \text{or } n_2 = n_1 \frac{\Phi_1}{\Phi_2}$$

$$\therefore n_2 = 800 \times \frac{100}{80} = 1000 \text{ r.p.m.}$$

Homework No.2 solution

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 810488 + 610383 + 610384

A six – poles DC machine has a flux per pole of 15 mwb. The armature has 536 conductors connected in wave winding . The DC machine runs at 1050 r.p.m and it delivers a rated armature current of 22.5 A to a load connected to it's terminals. Calculate :

- Machine constant K_e .
- Generated voltage E .
- Conductor current I_c .
- Electromagnetic torque T_e .
- Power P_d developed by the armature .

$$\left\{ \begin{array}{l} P = 6, a = 2 \\ \Phi = 15 \times 10^{-3} \text{ wb} \\ Z = 536 \\ n = 1050 \text{ r.p.m} \\ I_a = 22.5 \text{ A} \end{array} \right.$$

Solution:

$$\begin{aligned} \text{a) } K_e &= \left(\frac{P}{60} \right) \left(\frac{Z}{a} \right) \\ &= \frac{6}{60} \times \frac{536}{2} = \underline{\underline{26.8}} \end{aligned}$$

$$\begin{aligned} \text{b) } E &= K_e n \Phi \\ &= 26.8 \times 1050 \times 15 \times 10^{-3} = \underline{\underline{422.1 \text{ V}}} \end{aligned}$$

$$\text{c) } I_c = \frac{I_a}{a} = \frac{22.5}{2} = \underline{\underline{11.25 \text{ A}}}$$

$$\begin{aligned} \text{d) } T_e &= \frac{ZP}{2\pi a} I_a \Phi = \frac{536 \times 6}{2\pi \times 2} \times 22.5 \times 15 \times 10^{-3} \\ &= \underline{\underline{86.37 \text{ N.m}}} \end{aligned}$$

$$\text{e) } P_d = E I_a = 422.1 \times 22.5 = \underline{\underline{9497.25 \text{ W}}}$$

Small Electrical Machines

Dr. M. T. Lazim

Tutorial Examples (1)

1. A small 8-pole d.c. machine has an armature wound with 480 conductors. The magnetic flux and the speed are such that the average e.m.f generated in each conductor is 2.2V, and each conductor is capable of carrying a full-load of 5A. Calculate the terminal voltage on load, the output current on full load and the total power generated on full load when the armature is connected:
- (a) Lap-connected, (b) wave-connected.

[Ans: (a) 132V, 40A, 5280W
(b) 528V, 10A, 5280W]

2. (a) Develop from first principles an expression for the e.m.f generated in a small d.c. machine.
(b) Calculate the e.m.f developed in the armature of a 2-pole small d.c. machine whose armature has 280 conductors and is revolving at 1000 rpm. The flux per pole is 0.03 Wb.

[Ans: 140V]

3. The armature of a 4-pole d.c. machine has 47 slots, each slot containing 6 conductors. The armature is wave-connected, and the flux per pole is 25 mWb. Calculate the speed should the machine run with to generate e.m.f of 250V.

[Ans: 1065 r.p.m.]

4. A 4-pole small d.c. machine has armature wound with 564 conductors and driven at 800 r.p.m. The flux per pole being 20 mWb. The current in each conductor is 6A. Calculate the total current, the e.m.f and the electrical power delivered by the armature if the conductors are connected
- (a) wave, (b) Lap.

[Ans: 12A, 301V, 3612W
24A, 150.5V, 3612W]

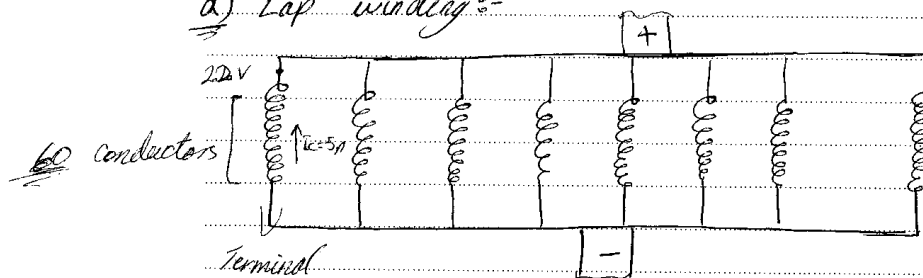
Tutorial Examples

1) Solution:

$$P = 8, \quad Z = 480, \quad E_c = 2.2 \text{ V}$$

$$I_c = 5 \text{ A},$$

a) Lap winding:-



8 Poles $\Rightarrow a = P = 8$
 { Lap winding }

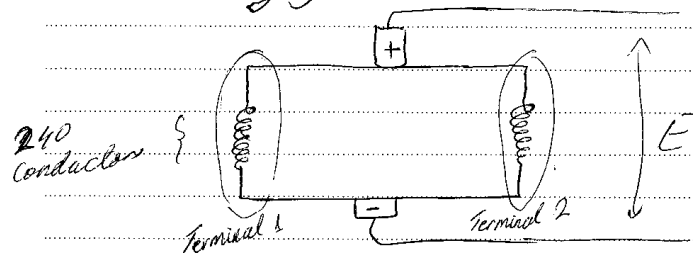
$$C_{\text{per pole}} = \frac{Z}{a} = \frac{480}{8} = 60 \text{ conductors in each pole.}$$

$$\bullet \quad E = 60 \times 2.2 = \underline{132 \text{ V}}$$

$$\bullet \quad I_a = I_c \times a = 5 \times 8 = \underline{40 \text{ A}}$$

$$\bullet \quad P_d = E I_a = 132 \times 40 = \underline{5280 \text{ W}}$$

b) wave winding:-



2-Poles $\Rightarrow a = P = 2$
 { wave-winding }

$$\frac{Z}{a} = \frac{480}{2} = 240 \Rightarrow 240 \text{ conductors in each pole.}$$

$$\bullet E = 240 \times 2.2 = \underline{528 \text{ V}}$$

$$\bullet \{ I_a = I_c \times a \} \Rightarrow I_a = 5 \times 2 = \underline{10 \text{ A}}$$

$$\bullet \{ P_d = E I_a \} \Rightarrow P_d = 528 \times 10 = \underline{5280 \text{ W}}$$

2 Solution:

a) $E = \left(\frac{P}{60}\right) \left(\frac{Z}{a}\right) n \Phi$ $E = \frac{\tau_e \omega}{I_a}$

b) $P = 2$ $Z = 280$ $n = 1000 \text{ r.p.m}$

$\Phi = 0.03 \text{ wb}$ $a = 2$

$E = \left(\frac{P}{60}\right) \left(\frac{Z}{a}\right) n \Phi$

$= \frac{2}{60} \times \frac{280}{2} \times 1000 \times 0.03$

$= \underline{140 \text{ V}}$

[3] Solution:

$P = 4$, 47 slots , 6 conductors in each slot

$a = 2$, $\Phi = 25 \times 10^{-3}$ wb

$E = 250$ V , $n = ???$, $Z = 47 \times 6 = 282$

$$E = \left(\frac{P}{60}\right) \left(\frac{Z}{a}\right) n \Phi$$

$$250 = \frac{4}{60} \times \frac{282}{2} \times 25 \times 10^{-3} \times [n]$$

$$\Rightarrow n = 1063.83 \text{ r.p.m}$$

[4] Solution: // a) wave b) lap

$$P = 4, Z = 564, n = 800 \text{ r.p.m.}$$

$$\Phi = 20 \times 10^{-3} \text{ wb}, I_c = 6 \text{ A}$$

~~Wave~~

a) • The total current $\Rightarrow I_a = I_c \cdot a = 6 \times 2 = \underline{12 \text{ A}}$

$$E = \left(\frac{P}{60}\right) \left(\frac{Z}{a}\right) n \Phi = \frac{4}{60} \times \frac{564}{2} \times 800 \times 20 \times 10^{-3} = \underline{300.8 \text{ V}}$$

$$P_d = E I_a = 300.8 \times 12 = \underline{3609.6 \text{ W}}$$

b) • The total current $\Rightarrow I_a = I_c \cdot a = 6 \times 4 = \underline{24 \text{ A}}$

$$E = \left(\frac{P}{60}\right) \left(\frac{Z}{a}\right) n \Phi = \frac{4}{60} \times \frac{564}{4} \times 800 \times 20 \times 10^{-3} = \underline{150.4 \text{ V}}$$

$$P_d = E I_a = 150.4 \times 24 = \underline{3609.6 \text{ W}}$$

Q1. An 8-pole armature is wound with 480 conductors. The magnetic flux and the speed are such that the average m.m.f generated in each conductor is 2.2V, and each conductor is capable of carrying a full-load current of 5 A. Calculate the terminal voltage on load, the output current on full load and the total power generated on full load when the armature is a) lap-connected and (b) wave-connected.

Solution:

(a) Lap-connected armature:

$$a = \text{no. of parallel paths} = \text{number of poles} = 8.$$

$$\text{no. of conductors / path} = \frac{480}{8} = 60.$$

$$\text{Terminal Voltage on no load} = (\text{e.m.f./conductor}) \times (\text{no. of conductor per path})$$

$$= 2.2 \times 60 = 132 \text{ V} = E.$$

$$\text{Output full load current} = (\text{full-load current per conductor}) \times (\text{no. of parallel paths})$$

$$= 5 \times 8 = 40 \text{ A} = I_a$$

$$\therefore \text{Total power } P_e = E \cdot I_a = 132 \times 40 = 5280 \text{ W}$$

(b) Wave-Connected armature:

$$a = 2$$

$$\text{no. of conductors / path} = \frac{480}{2} = 240.$$

$$\text{Terminal Voltage on no load} = 2.2 \times 240 = 528 \text{ V} = E$$

$$\text{Output current at full load} = 5 \times 2 = 10 \text{ A} = I_a$$

$$\text{Total power } P_e = E I_a = 528 \times 10 = 5280$$

**Small Electrical
Machines****Dr. M.T. Lazim****SERVOMECHANISM**

Servomechanism: is a device used to provide mechanical control at remote distance.

One of the most commonly used type of servomechanism (servo) is a position control system in which the output position, often the shaft position of a motor, must exactly follow the variation of the input position.

Applications:

- Radar tracking antenna position control.
- Airfoils and rudders in aircrafts and ships.
- Automated manufacturing facilities, such as positioning of drills, punches and spot welders.
- Robotics.
- Computer hardware systems.
- Satellite antenna position control in ground stations.

Servomechanism (servos) are commonly electrical, electrostatics or electronics in nature.

In electrical servos, the main driving element used is the electric motor which is considered as the primary means of creating mechanical forces. Therefore, according to the type of motor used (usually small motors), the servos are classified as:

- DC Servomotors (DC Servo).
- AC Servomotors (AC Servo).

1. DC - SERVOMOTORS

DC Servomotors are usually permanent magnet (PMDC) d.c. motors or separately excited type. A schematic diagram of a PMDC motor is shown in Fig.1

where:

V_a = armature supply voltage

E_a = armature induced back e.m.f

R_a = armature resistance

L_a = armature inductance

i_a = armature current

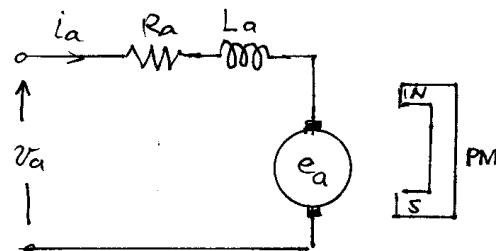


Fig.1
Armature Field

• Electrical Equations of the motor

The general electrical relationships of a servomotor is given by applying KVL on the armature circuit shown in Fig.1.

Let ω_m be the angular velocity of the motor:-

$$V_a = E_a + i_a R_a + L_a \frac{di_a}{dt}$$

$$E_a = K \cdot \Phi \cdot \omega_m$$

$$= K' \cdot \omega_m$$

where $K = \frac{60}{2\pi} K_e = 9.55 K_e = K_T$

$K' = K \cdot \Phi$ for constant flux.

• Mechanical Equations of the Motor:

The mechanical torque is given by:

$$T_e = K \cdot \Phi \cdot i_a$$

$$= K' \cdot i_a$$

for constant flux

Let :

θ_m = Angular position or displacement (rad)

$\omega_m = \frac{d\theta_m}{dt}$: Angular velocity (speed) rad/sec.

$\alpha = \frac{d\omega_m}{dt}$: Angular acceleration

$$= \frac{d^2\theta_m}{dt^2} \quad \text{rad/sec}^2.$$

J = Rotational mass (moment of inertia).

Notes: • In linear mechanics mass = m

• In rotational mechanics mass = J

$$\begin{aligned} \bullet \text{ Kinetic Energy} &= \frac{1}{2} m v^2 \quad (\text{linear mechanics}) \\ &= \frac{1}{2} J \omega_m^2 \quad (\text{rotational mechanics}) \end{aligned}$$

$$\begin{aligned} \bullet \text{ Torque} &= F \cdot d \quad (\text{linear mechanics}) \\ &= J \cdot \alpha \quad (\text{rotational mechanics}). \\ &= J \cdot \frac{d\omega_m}{dt} \end{aligned}$$

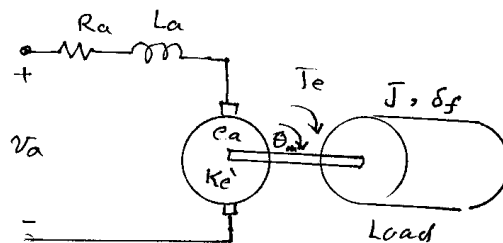
DC motor + load
Representation :

The motor torque T_e is balanced by the inertia torque $J \frac{d\omega_m}{dt}$ + the friction torque $\delta_f \omega_m$, where δ_f is the total

friction coefficient (viscous friction) of the motor and load rotating parts. Thus:

$$T_e = J \frac{d\omega_m}{dt} + \delta_f \omega_m$$

This is called the equation of motion of electric motor.



Consider the d.c. servo system shown in Fig.1.
The operation of the system is as follows:

- A pair of potentiometers act as an error measuring device.
They convert the input and output positions into proportional electric signals.
- The command input signal determines the angular position r of the wiper arm of the input potentiometer. The input angular position r is the reference input to the system, and the electric potential of the arm is proportional to the angular position of the arm.
- The output shaft position determines the angular position C of the output wiper arm of the output potentiometer.
- The difference between the input angular position r and the output angular position C is the error signal(e), or:

$$e = r - C$$

- The potential difference $e_r - e_c = e_v$ is the error voltage, where

$$e_r \propto r \quad \text{and} \quad e_c \propto C.$$

$$\text{that is } e_r = k_0 r \quad \text{and} \quad e_c = k_0 C$$

where k_0 is the proportionality constant.

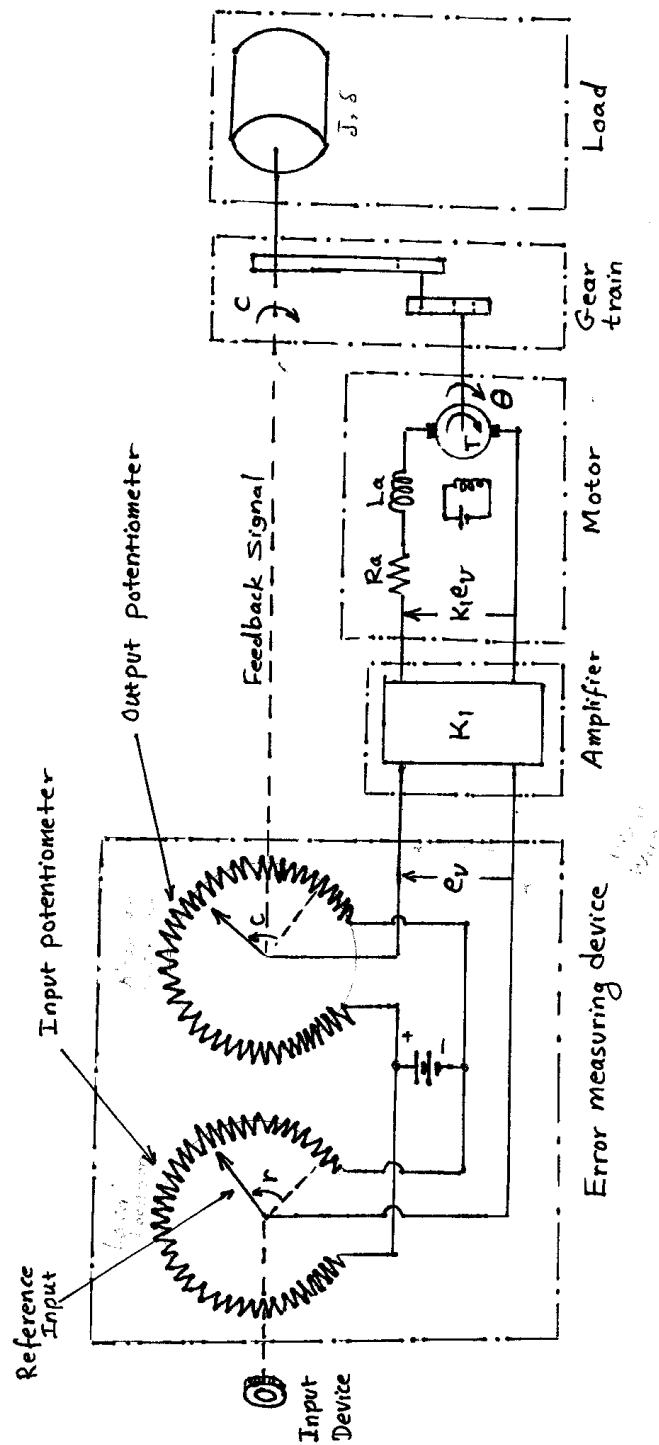


Fig.1 Schematic diagram of d.c. Servo system

- e_r will be amplified by amplifier whose gain is K_1 .
- The output voltage of this amplifier is applied to the armature circuit of the d.c. motor.
- If an error exist, the motor develops a torque to rotate the output load in such away to reduce the error to zero.

$$T = K' i_a$$

- When the armature is rotating, back e.m.f will be generated:

$$e_b = K' \omega_m = K' \frac{d\theta}{dt}$$

For the d.c. motor + Load equations:

The differential equation for the armature circuit is:

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a$$

$$L_a \frac{di_a}{dt} + R_a i_a + K' \frac{d\theta}{dt} = K_1 e_r$$

The equation for the torque equilibrium is

$$J \frac{d^2\theta}{dt^2} + \delta_f \frac{d\theta}{dt} = T = K' i_a$$

Example.1: A small separately excited d.c. motor is directly coupled to a pure-inertia, friction-less load ($\delta_f = 0$), as shown in Fig.A. With the motor at rest, the constant direct voltage V_a is suddenly applied on its armature terminals.

- Obtain an expression of the armature current as a function of time.
- Obtain an expression for the angular velocity of the shaft as a function of time.
- Consider that the resistance $R_a = 34 \Omega$, the moment of inertia $J = 5.85 \times 10^{-5} \text{ kg}\cdot\text{m}$ and the constants $K' = 0.328 \text{ V/rad/sec. or N}\cdot\text{m/A}$. Determine the time for the motor to come within approximately 80 percent of its final speed. Neglect the armature inductance L_a .

Solution
By KVL:

$$V_a = e_b + R_a i_a + L_a \frac{di_a}{dt}$$

$$= e_b + R_a i_a \quad \dots (1)$$

$$\therefore V_a = K' \omega_m + R_a i_a$$

$$T = K' i_a = J \frac{d\omega_m}{dt} + \delta_f \omega_m$$

$$\therefore K' i_a = J \frac{d\omega_m}{dt} \quad \dots (2)$$

From eq.(1)

$$\frac{dV_a}{dt} = \frac{de_b}{dt} + R_a \frac{di_a}{dt}$$

$$0 = K' \frac{d\omega_m}{dt} + R_a \frac{di_a}{dt} \quad \dots (3) \quad \text{since } e = K' \omega$$

$$\text{From eq.(2)} \quad \frac{d\omega_m}{dt} = \frac{K'}{J} i_a \quad \dots (4)$$

Substitute (4) in (3) yields

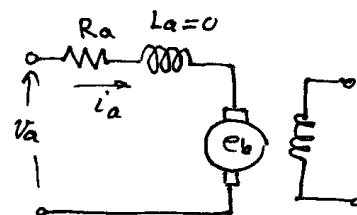


Fig-A

$$0 = K' \cdot \frac{K'}{J} i_a + R_a \frac{di_a}{dt}$$

$$0 = \frac{K'^2}{J} i_a + R_a \frac{di_a}{dt}$$

$$\text{or } \frac{di_a}{dt} + \frac{K'^2}{J R_a} i_a = 0$$

This is a first order differential equation with initial conditions:

At $t = 0^+$, $\omega_m = 0$, $e_b = 0$, and $i_a = \frac{V_a}{R_a}$ (from eq.(1)).

$$\text{We have } \left[\begin{array}{l} \frac{dx}{dt} + \alpha x = 0 \\ x = x(0) e^{-\alpha t} \end{array} \right. \quad \text{solution for F.O.D.E.}$$

$$\text{Here } x(0) = i_a(0) = \frac{V_a}{R_a} \quad \& \quad \alpha = \frac{K'^2}{J R_a}$$

$$\therefore i_a = \frac{V_a}{R_a} e^{-\frac{K'^2}{J R_a} t}$$

b. From eq. 4 :

$$i_a = \frac{J}{K'} \frac{d\omega_m}{dt} \quad \dots \dots (5)$$

Substitute (5) into (1) yields :

$$V_a = K' \omega_m + \frac{R_a J}{K'} \frac{d\omega_m}{dt}$$

$$\text{or } \frac{V_a}{K'} = \omega_m + \frac{R_a J}{K'^2} \frac{d\omega_m}{dt}$$

$$\text{or } \frac{d\omega_m}{dt} + \frac{K'^2}{J R_a} \omega_m = \frac{V_a}{R_a} \cdot \frac{K'}{J} = \frac{V_a}{K'} \cdot \frac{K'^2}{R_a J}$$

The solution of this D.E is

$$\omega_m = \frac{V_a}{K'} \left(1 - e^{-\frac{K'^2}{J R_a} t} \right) = \frac{V_a}{K'} \left(1 - e^{-\frac{t}{\tau}} \right) \quad \dots (6)$$

c. From Eq. 6

$$\tau = \frac{1}{\frac{K'^2}{J R_a}} = \frac{(5.85)(10)^{-5} (34)}{(0.328)^2} = 0.0185 \text{ Sec.}$$

11

we have the equation of the motor speed

$$\omega_m = \frac{V_a}{K'} (1 - e^{-\frac{K'^2}{JR_a} t}) = \omega_\infty (1 - e^{-\frac{t}{\tau}})$$

where ω_∞ = Final speed. $= \frac{V_a}{K'}$

$$\frac{80}{100} \omega_\infty = \omega_\infty (1 - e^{-\frac{t}{0.0185}})$$

$$0.80 = 1 - e^{-\frac{t}{0.0185}}$$

$$-1 + 0.80 = -e^{-\frac{t}{0.0185}}$$

$$-0.2 = -e^{-\frac{t}{0.0185}}$$

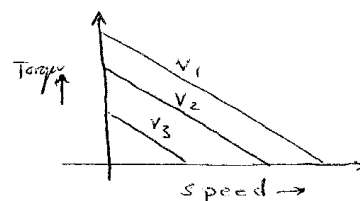
$$\ln 0.2 = -\frac{t}{0.0185}$$

$$-1.6 = -\frac{t}{0.0185}$$

$$\therefore t = 1.6 \times 0.0185 = 29 \text{ ms.}$$

DC Servomotors Features:

- ① DC servomotors ranges from few watts to several hundred Watts.
- ② Most of high power servomotors are d.c. servo.
- ③ Torque-Speed Characteristics of d.c. servomotors are approximately ideal (straight lines)



To find the angle of displacement:

$$\frac{d\omega_m}{dt} + \frac{K'^2}{JRa} \omega_m = \frac{V_a}{Ra} \cdot \frac{K'}{J}$$

$$\text{let } a = \frac{K'^2}{JR} \quad \text{and } b = \frac{V_a}{Ra} \cdot \frac{K'}{J}$$

$$\frac{d^2\theta}{dt^2} + a \frac{d\theta}{dt} = b$$

Using Laplace solution:

$$s^2 \theta(s) + a s \theta(s) = b$$

$$\theta(s) (s^2 + as) = b$$

$$\theta(s) = \frac{b}{s^2 + as} = \frac{b}{s(s+a)}$$

$$\frac{b}{s(s+a)} = \theta(s) = \frac{A_1}{s} + \frac{A_2}{(s+a)}$$

$$A_1 = \frac{b}{a}, \quad A_2 = -\frac{b}{a}$$

$$\theta(s) = \frac{b}{a} \left[\frac{1}{s} - \frac{1}{s+a} \right]$$

Take Laplace inverse

$$\theta(t) = \frac{b}{a} [1 - e^{-at}]$$

$$\frac{b}{a} = \frac{\frac{V_a}{Ra} \cdot \frac{K'}{J}}{\frac{K'^2}{JRa}} = \frac{V_a}{K'}$$

$$\therefore \theta(t) = \frac{V_a}{K'} \left[1 - e^{-\frac{K'^2}{JRa} t} \right]$$

$$\begin{array}{l} s \rightarrow 0 \\ s \rightarrow -a \end{array}$$

Small Electrical Machines

Dr. Mohammed Tawfeeq

SERVOMECHANISM - PART 2

II. AC SERVOMOTORS

For low power servo systems (from 1W - 100 W), two-phase induction motors are often used.

The two-phase induction motor consists of: a

- Stator has two windings displaced 90° in space.
- Rotor (Solid or squirrel-cage or Drag-Cup rotor type)

For approximate evaluation of the motor performance it is assumed that

- (1) The torque-speed characteristics are assumed linear with negative slope.
- (2) The stalling torque is assumed to be linearly proportional to the r.m.s control winding voltage (V_2).

Hence:

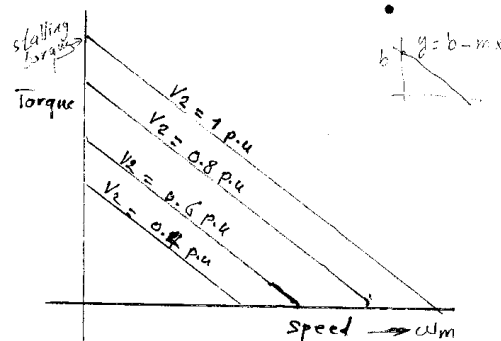
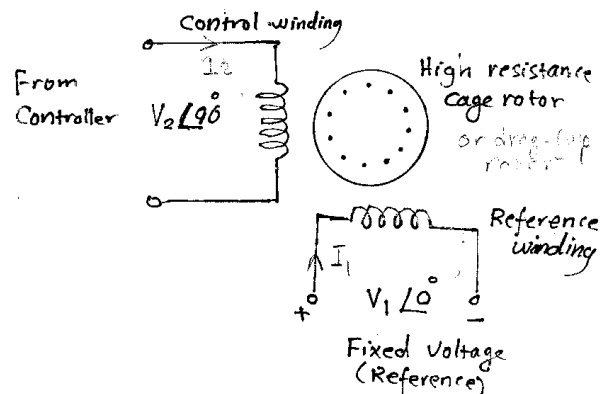
$$T = K_1 V_2 - K_2 \omega_m$$

where:

ω_m = shaft mechanical velocity.

V_2 = rms value of the control voltage.

K_1, K_2 are constants.

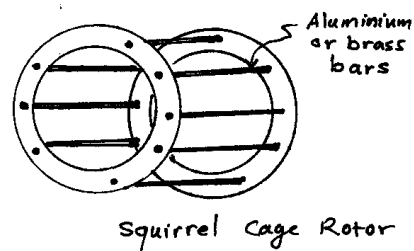


Typical speed-torque characteristic of two-phase induction motor with high resistance cage rotor.

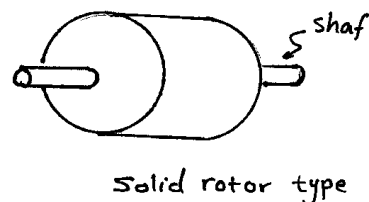
Types of Two-phase Induction Motors

There are three types of two-phase induction motors used in AC servo systems:

- a) Squirrel - Cage rotor type (rotor conductors are made of brass, copper or aluminium)



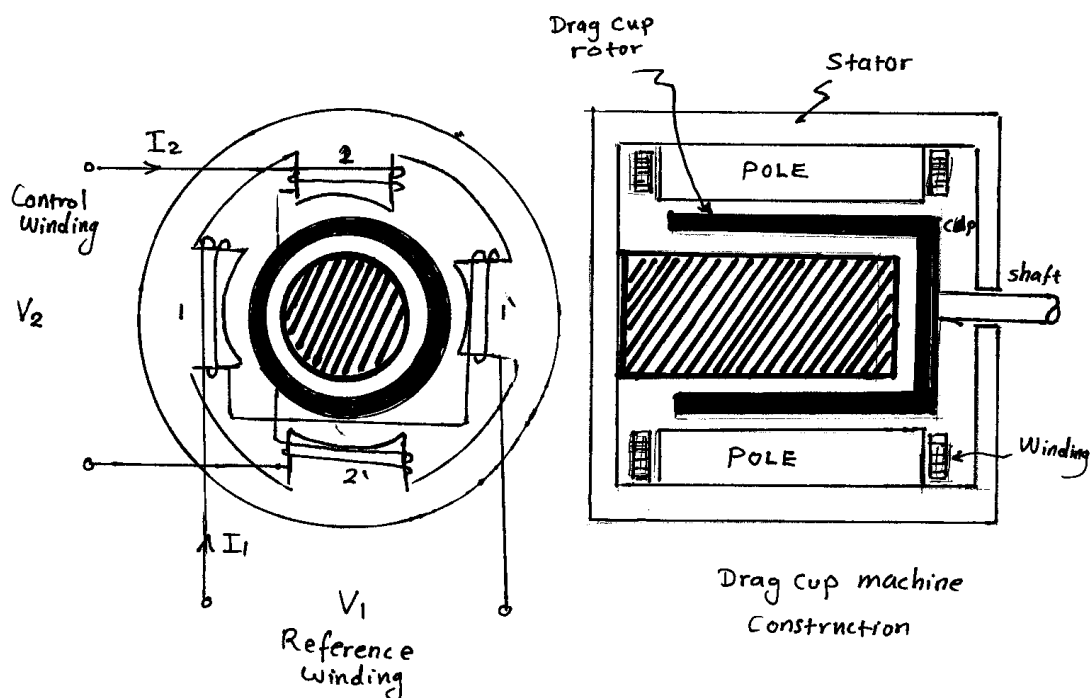
- b) Solid rotor type (rotor made of solid ferromagnetic materials)



- c) drag cup (drum) rotor type (rotor made of thin aluminium or copper cylinder) the rotor has a small moment of inertia.

The drag cup rotor induction motor are widely used in AC servo systems since they have the following advantages:

1. Very small time constant due to small inertia J . ($\tau_m = \frac{J}{K_s}$).
2. Need no maintenance
3. Easy to revolve (very small friction).



Such type of motor has two windings (1-1') and (2-2') shifted by 90° electrical in space.

One winding (1-1') is supplied by constant voltage V_1 , and the second winding (2-2') supplied by the control voltage V_2 having variable magnitude and phase. According to the phase shift and magnitude of the two currents I_1 and I_2 , a resulting magnetic field is created in either circular or elliptical pattern.

- Circular field is obtained when $|I_1| = |I_2|$ and shifted by $\pm 90^\circ$.
- Elliptical field is produced when :
 - a) $|I_1| \neq |I_2|$ and phase shift is $\pm 90^\circ$
 - b) $|I_1| = |I_2|$ phase shift is different from $\pm 90^\circ$.

In case of circular field maximum revolution is obtained.



Example:

A two-phase servomotor has rated voltage applied to its reference phase winding. The torque-speed characteristic of the motor with $V_2 = 115\text{ V}$, 60 Hz applied to its control phase is shown in Fig. A. The moment of inertia of the motor and load $J = 10^{-5} \text{ Kg}\cdot\text{m}^2$, and the viscous friction of the load is negligible.

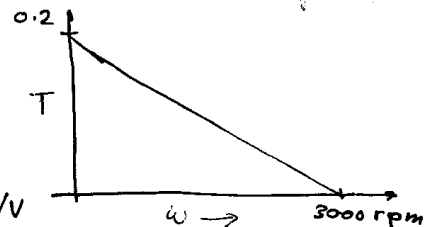
- obtain the transfer function between shaft position θ and control voltage V_2 .
- obtain an expression for the shaft position due to the application of a step voltage $V_2 = 115\text{ V}$ to the control phase winding. Obtain $\theta(t)$ for $t > 0$.

Solution

Since $T = K_1 V_2 - K_2 \omega_m$

$$K_1 = \left. \frac{T}{V_2} \right|_{\omega_m = \text{constant}}$$

$$= \left. \frac{0.2}{115} \right|_{\omega_m = 0} = 0.00174 \text{ N}\cdot\text{m/V}$$



$$K_2 = \left. \frac{T}{\omega_m} \right|_{V_2 = \text{constant}} = \frac{0.2}{3000 \times \frac{2\pi}{60}} = 6.366 \times 10^{-4} \text{ N}\cdot\text{m/rad/sec.}$$

$$K_3 = K_2 + \delta_f = K_2 + 0 = K_2 = 6.366 \times 10^{-4} \text{ N}\cdot\text{m/rad/sec.}$$

$$J = 10^{-5} \text{ Kg}\cdot\text{m}^2$$

$$T_m = \frac{J}{K_3} = \frac{10^{-5}}{6.366 \times 10^{-4}} = 15.71 \times 10^{-3} \text{ sec.}$$

$$\frac{K_1}{K_3} = \frac{0.00174}{6.366 \times 10^{-4}} = 2.733$$

$$\frac{\Theta(s)}{V_2(s)} = \frac{K_1/K_3}{s(1+sT_m)} = \frac{2.733}{s(1+0.01571s)}$$

$$V_2(s) = \frac{115}{s}$$

$$\frac{K_1}{K_3} V = 2.733 \times 115 = 314.3$$

$$\frac{K_1 V}{K_3} T_m = 314.3 \times 0.01571 = 4.94$$

but we have:

$$\Theta(t) = \frac{K_1 V}{K_3} t$$

$$\therefore \Theta(t) = 314.3 t$$

(c) What is the angle Θ in degrees after time of 5 msec.

$$\Theta(t) = 314.3 \times 5 \times 10^{-3} = 1.5715 \text{ rad.}$$

$$= \frac{1.5715}{3.14} \times 180 = \underline{\underline{90^\circ}}$$

Speed and position equations

The motor equation of motion is generally given by:

$$T = J \frac{d\omega_m}{dt} + \delta_f \omega_m$$

For two-phase motor,

$$T = K_1 V_2 - K_2 \omega_m$$

$$\therefore J \frac{d\omega_m}{dt} + \delta_f \omega_m = K_1 V_2 - K_2 \omega_m$$

$$J \frac{d\omega_m}{dt} + (\delta_f + K_2) \omega_m = K_1 V_2 \quad \dots \dots \dots (1) \quad \begin{matrix} K_2 = \text{slope of} \\ \text{torque-speed} \\ \text{ch/s} \end{matrix}$$

In Laplace form:

$$J s \omega(s) + K_3 \omega(s) = K_1 V_2(s)$$

$$\omega(s) (J s + K_3) = K_1 V_2(s)$$

$$\therefore \frac{\omega(s)}{V_2(s)} = \frac{K_1}{(J s + K_3)} = \frac{K_1/K_3}{(\frac{J}{K_3} s + 1)} \quad \text{where } K_3 = \delta_f + K_2$$

$$\text{Let } T_m = \frac{J}{K_3} \quad (\text{mechanical time constant})$$

$$\therefore \boxed{\frac{\omega(s)}{V_2(s)} = \frac{K_1/K_3}{(1 + T_m s)}} \quad \dots \dots \dots (2)$$

Now if θ = displacement of load $\therefore \frac{d\theta}{dt} = \omega_m$

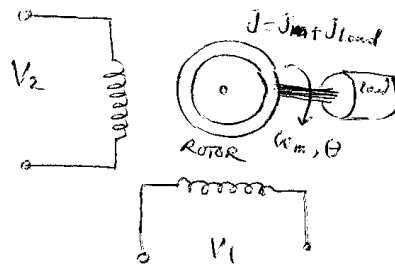
From equation (1):

$$J \frac{d^2\theta}{dt^2} + K_3 \frac{d\theta}{dt} = K_1 V_2 \quad \dots \dots \dots (3)$$

$$\text{or } J s^2 \theta(s) + K_3 \theta(s) = K_1 V_2(s)$$

$$\theta(s) [J s^2 + K_3 s] = K_1 V_2(s)$$

$$\therefore \boxed{\frac{\theta(s)}{V_2(s)} = \frac{K_1}{J s^2 + K_3 s} = \frac{K_1/K_3}{(T_m s^2 + s)} = \frac{K_1/K_3}{s(1 + T_m s)}} \quad \dots \dots \dots (4)$$



J_m = motor inertia
 J_L = load inertia
 δ_f = viscous friction force
 K_2 = slope of torque-speed ch/s

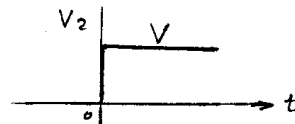
AC SERVOMOTOR ANALYSIS :

Time response for a step change in the control voltage:

For a step change in the control voltage

$$\text{Let: } V_2(s) = \frac{V}{s}$$

$$\omega_m(s) = \frac{K_1/K_3}{1+sT_m} \cdot V_2(s) = \frac{K_1/K_3}{1+sT_m} \cdot \frac{V}{s}$$



Let:

$$\omega_m(s) = \frac{(K_1/K_3)V}{s(\frac{1}{T_m} + sT_m)} = \frac{A_1}{s} + \frac{A_2}{s + \frac{1}{T_m}} \quad \dots\dots(1)$$

1. Multiply both side of eq.(1) by s:

$$\frac{(K_1/K_3)V}{(s + \frac{1}{T_m})T_m} = A_1 + \frac{A_2 s}{s + \frac{1}{T_m}}$$

$$\text{Let } s = 0$$

$$\therefore (K_1/K_3)V = A_1$$

2. Multiplying both side of eq.(1) by $(s + \frac{1}{T_m})$

$$\frac{(K_1/K_3)V}{s T_m} = \frac{A_1}{s} (s + \frac{1}{T_m}) + A_2$$

$$\text{Let } s = -\frac{1}{T_m}$$

$$\frac{(K_1/K_3)V}{(-\frac{1}{T_m})T_m} = A_2 \quad \therefore A_2 = -\frac{K_1}{K_3} V$$

Substitute A_1 & A_2 into eq.(1):

$$\omega_m(s) = \frac{K_1}{K_3} V \left(\frac{1}{s} - \frac{1}{s + \frac{1}{T_m}} \right)$$

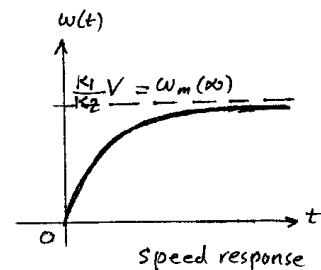
Take Laplace inverse:

$$\mathcal{L}^{-1}(\omega_m(s)) = \omega_m(t) = \frac{K_1 V}{K_3} \left[\mathcal{L}^{-1} \frac{1}{s} - \mathcal{L}^{-1} \frac{1}{s + \frac{1}{T_m}} \right]$$

$$\therefore \omega_m(t) = \frac{K_1 V}{K_3} (1 - e^{-\frac{t}{T_m}})$$

For steady-state $t \rightarrow \infty$

$$\omega_m(\infty) = \frac{K_1 V}{K_3}$$



The position $\theta(s)$ due to step voltage is:

$$\Theta(s) = \frac{K_1/K_3}{s(1+\tau_m s)} \cdot \frac{V}{s} = \frac{K_1/K_3 V}{s^2(s+\frac{1}{\tau_m})\tau_m}$$

Solution of this equation:

$$\frac{(K_1/K_3)V}{s^2(s+\frac{1}{\tau_m})\tau_m} = \frac{A_1}{s^2} + \frac{A_2}{s} + \frac{A_3}{(s+\frac{1}{\tau_m})} = \frac{A_1(s+\frac{1}{\tau_m}) + A_2s(s+\frac{1}{\tau_m}) + A_3s^2}{s^2(s+\frac{1}{\tau_m})}$$

$$\therefore \frac{(K_1/K_3)V}{\tau_m} = A_1(s+\frac{1}{\tau_m}) + A_2s(s+\frac{1}{\tau_m}) + A_3s^2$$

① let $s=0$

$$A_1 = (K_1/K_3)V$$

② let $s = -\frac{1}{\tau_m}$

$$\frac{(K_1/K_3)V}{\tau_m} = A_3\left(-\frac{1}{\tau_m}\right)^2 \quad \text{or} \quad A_3 = (K_1/K_3)V\tau_m$$

③ let $s=1$

$$\frac{(K_1/K_3)V}{\tau_m} = \frac{K_1V}{K_3}\left(1+\frac{1}{\tau_m}\right) + A_2\left(1+\frac{1}{\tau_m}\right) + \frac{K_1V}{K_3}\tau_m$$

$$A_2\left(\frac{\tau_m+1}{\tau_m}\right) = \frac{K_1V}{K_3}\left[\frac{1}{\tau_m} - \left(1+\frac{1}{\tau_m}\right) - \tau_m\right] = \frac{K_1V}{K_3}\left[\frac{1}{\tau_m} - 1 - \frac{1}{\tau_m} - \tau_m\right]$$

$$A_2(1+\tau_m) = -\frac{K_1V}{K_3}[1+\tau_m]\tau_m$$

$$\therefore A_2 = -\frac{K_1V\tau_m}{K_3}$$

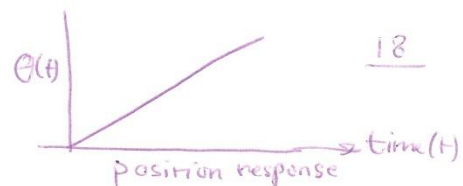
Hence $\Theta(s) = \frac{K_1V}{K_3} \cdot \frac{1}{s^2} - \frac{K_1V\tau_m}{K_3} \left(\frac{1}{s}\right) + \frac{K_1V\tau_m}{K_3} \frac{1}{(s+\frac{1}{\tau_m})}$

In time domain

$$\Theta(t) = \frac{K_1V}{K_3} t - \frac{K_1V\tau_m}{K_3} + \frac{K_1V\tau_m}{K_3} e^{-\frac{t}{\tau_m}}$$

In most applications the two-phase motor of drag-cup rotor has negligible rotor inertia, i.e. $J \approx 0$ hence, $\tau_m = \frac{J}{K_3} \approx 0$

$$\therefore \Theta(t) \approx \frac{K_1V}{K_3} t$$



problem

A simple antenna position control servomechanism shown in Fig.1 using two-phase d.c. servo motor. The differential equation describing the system is given by:

$$0.05 \frac{d^2\theta}{dt^2} + 0.5 \frac{d\theta}{dt} + 1.2\theta = 0.05 \theta_{ref}$$

where θ is the position of the antenna and θ_{ref} is the command angle of the input potentiometer.

Let the potentiometer transduces give a voltage of 1 Volt per radian position,

- Derive the transfer function of the system
- For a step change in the command angle of $180^\circ (= \pi \text{ radians})$, find time response for the angular position of the antenna.

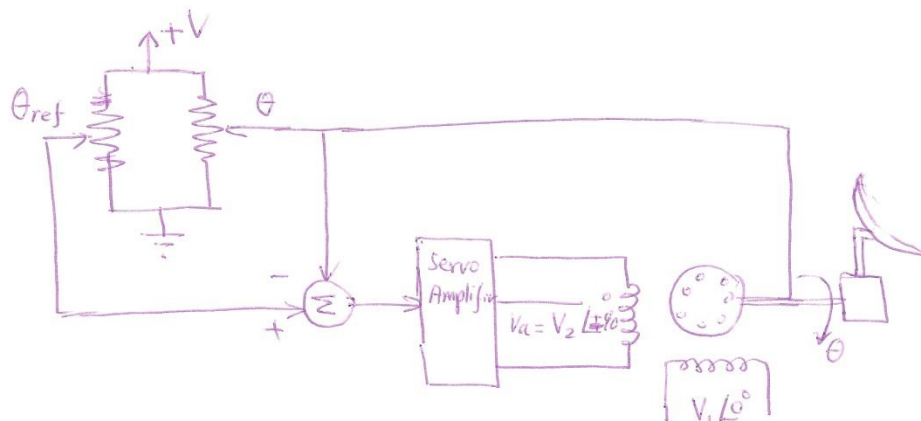


Fig.1

solution :

$$(a) \quad 0.05 \frac{d^2\theta}{dt^2} + 0.5 \frac{d\theta}{dt} + 1.2\theta = 0.05 \theta_{ref}.$$

$$\frac{d^2\theta}{dt^2} + 10 \frac{d\theta}{dt} + 24\theta = \theta_{ref}.$$

In Laplace form :

$$s^2 \theta(s) + 10s \theta(s) + 24 \theta(s) = \theta_{ref}(s).$$

$$\theta(s) (s^2 + 10s + 24) = \theta_{ref}(s).$$

$\therefore$ The transfer function is :

$$\frac{\theta(s)}{\theta_{ref}(s)} = \frac{1}{s^2 + 10s + 24}$$

$$(b) \text{ A } \pi \text{ step change of } \theta_{ref}(s) \Rightarrow \frac{\pi}{s}.$$

$$\therefore \theta(s) = \frac{\pi}{s(s^2 + 10s + 24)} = \frac{\pi}{s(s+4)(s+6)}.$$

$$\text{or } \theta(s) = \frac{\pi}{s(s+4)(s+6)} = \frac{A_1}{s} + \frac{A_2}{(s+4)} + \frac{A_3}{(s+6)}$$

$$\pi = A_1(s+4)(s+6) + A_2s(s+6) + A_3(s+4)s$$

$$(1) \text{ let } s=0 \\ \pi = A_1 \times 4 \times 6 = 24A_1 \Rightarrow A_1 = \frac{\pi}{24}$$

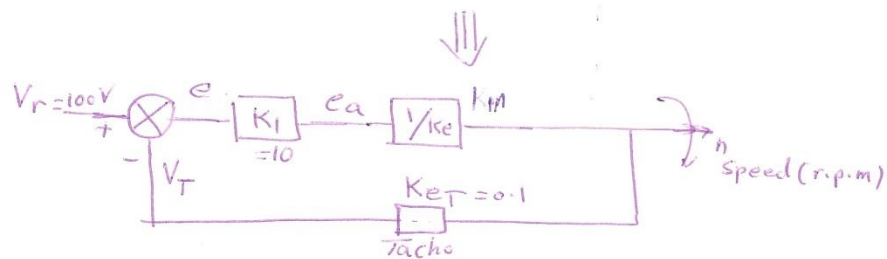
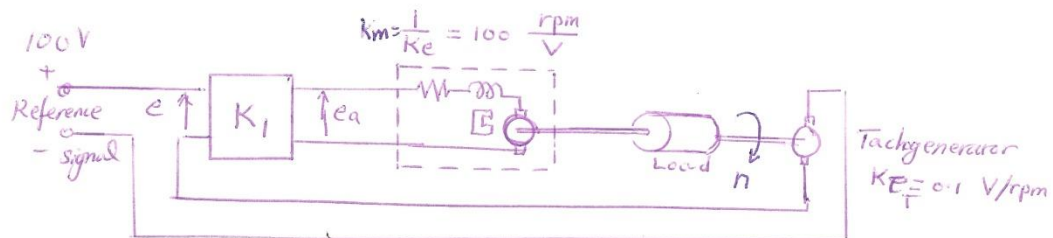
$$(2) \text{ let } s=-4 \\ \pi = (0) + A_2(-4)(2) = -8A_2 \Rightarrow A_2 = -\frac{\pi}{8}$$

$$(3) \text{ let } s=-6 \\ \pi = (0) + (0) + A_3(-6)(-2) = +12A_3 \Rightarrow A_3 = \frac{\pi}{12}$$

$$\therefore \theta(s) = \frac{\pi}{24s} - \frac{\pi}{8(s+4)} + \frac{\pi}{12(s+6)}$$

$$\text{or } \theta(t) = \frac{\pi}{24} - \frac{\pi}{8} e^{-4t} + \frac{\pi}{12} e^{-6t} \\ = \frac{\pi}{24} \left[1 - 3e^{-4t} + 2e^{-6t} \right].$$

Remote position control servomechanism Stabilized by velocity feedback system



$$V_T = V_{\text{Tacho}} = K_{E_T} n = 0.1 n$$

$$e_a = \text{Amplifier output} = K_1 e = \text{armature voltage}$$

$$e = \text{error} = V_r - V_T = 100 - 0.1n$$

$$n = \text{Motor speed} = e_a \times \left(\frac{1}{K_e}\right) = K_1 (100 - 0.1n) \times 100$$

$$\text{or } n = 10(100 - 0.1n) \times 100 = 100000 - 100n$$

$$\therefore n(101) = 100000 \text{ and } n = \frac{100000}{101} = 990.1 \text{ rpm}$$

Problem 1:

Derive the differential equation for a position control servomechanism stabilized by velocity feedback and show that, if there is a permanent friction force in the system represented by coefficient of friction δ_f , the equation is of the form:

$$J \frac{d^2\theta_c}{dt^2} + (\delta_f + F) \frac{d\theta_c}{dt} + K \theta_c = K \theta_R$$

where F is the velocity damping coefficient $= \frac{K_e K_T K_A}{R_a}$

Solution: -

For dc servo system

$$e_v = K_p \theta_R - K_p \theta_c \\ = K_p (\theta_R - \theta_c) \quad \dots (1)$$

K_p is input and output potentiometer constant V/rad.

For DC Motor:

$$J \frac{d^2\theta_c}{dt^2} + \delta_f \frac{d\theta_c}{dt} = T = k_T i_a \quad \dots (2) \quad \text{Equation of motion}$$

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a \quad \dots (3) \quad , e_a = K_A e_v$$

if L_a is small $\rightarrow 0$.

$$R_a i_a + K_e \frac{d\theta_c}{dt} = K_p (\theta_R - \theta_c) K_A$$

$$i_a = \frac{-K_e}{R_a} \frac{d\theta_c}{dt} + \frac{K_p K_A}{R_a} \theta_R - \frac{K_p K_A}{R_a} \theta_c \quad \dots (4)$$

Substitute eq. (4) into (2):

$$J \frac{d^2\theta_c}{dt^2} + \delta_f \frac{d\theta_c}{dt} = \frac{-K_T K_e}{R_a} \frac{d\theta_c}{dt} + \frac{K_T K_p K_A}{R_a} \theta_R - \frac{K_T K_p K_A}{R_a} \theta_c$$

$$\text{Let } K = \frac{K_T K_p K_A}{R} \quad , \quad F = \frac{K_T K_e}{R_a}$$

$$\therefore J \frac{d^2\theta_c}{dt^2} + (\delta_f + F) \frac{d\theta_c}{dt} + K \theta_c = K \theta_R$$

$$J s^2 \theta_c(s) + B s \theta_c(s) + K \theta_c(s) = K \theta_R(s) \quad (B = \delta_f + F)$$

$$\theta_c(s) (J s^2 + B s + K) = K \theta_R(s)$$

$$\frac{\theta_c(s)}{\theta_R(s)} = \frac{K}{J s^2 + B s + K} = \frac{K/J}{(s^2 + \frac{B}{J} s + \frac{K}{J})}$$

problem 2: A particular servosystem for which the above is the equation of motion, has the following constants:

$$\text{Undamped natural frequency} = \frac{\sqrt{6}}{2\pi} \text{ Hz}$$

$$\text{Coefficient of viscous friction } \delta f = 8 \text{ N.m per rad/sec.}$$

$$\text{Velocity damping coefficient } F = 12 \text{ N.m per rad/sec.}$$

$$\text{Moment of inertia } J = 4 \text{ Kg.m}^2.$$

Determine the angle turned through by the output shaft 100 ms after θ_R is changed from 0 to 0.6 radians.

Solution:

$$\frac{d^2\theta_c}{dt^2} + \frac{\delta f + F}{J} \frac{d\theta_c}{dt} + \frac{K}{J} \theta_c = \frac{K}{J} \theta_R$$

$$\frac{d^2\theta_c}{dt^2} + 2\alpha \frac{d\theta_c}{dt} + \omega_0^2 \theta_c = \frac{K}{J} \theta_R$$

$$\text{Where } 2\alpha = \frac{\delta f + F}{J} = \frac{8 + 12}{4} = 5$$

$$\omega_0 = \sqrt{\frac{K}{J}} = \sqrt{\frac{K}{4}} = \frac{\sqrt{6}}{2\pi}$$

$$\frac{K}{4} = \frac{6}{4\pi^2} \quad \therefore K = \frac{6}{\pi^2}$$

$$\therefore \omega_0 = \sqrt{\frac{6}{4\pi^2}} = \frac{\sqrt{6}}{2\pi} = 0.41$$

$$2\alpha = 5 \quad \therefore \alpha = 2.5$$

$\alpha > \omega_0$, roots are real and the solution is overdamped.

$$\theta_c(t) = A_1 e^{s_1 t} + A_2 e^{s_2(t)}$$

$$\text{where } s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$s_1 = -2.5 + \sqrt{(2.5)^2 - (0.41)^2} = -2.5 + 2.48 = -0.002$$

$$s_2 = -2.5 - 2.48 = -4.98$$

$$\therefore \theta_c(t) = A_1 e^{-0.002t} + A_2 e^{-4.98t}$$

Small (miniature) Machines

1. Permanent-Magnet Machines

- Very small dc motors are often made with permanent magnet (PM) fields. The stator wound of conventional DC motors are replaced by permanent magnets. The rotor has a conventional DC armature winding with commutator segments and brushes. The 2-pole machine shown in Fig.1 has "annular" magnet that may be cross-magnetized or it may be radially magnetized.

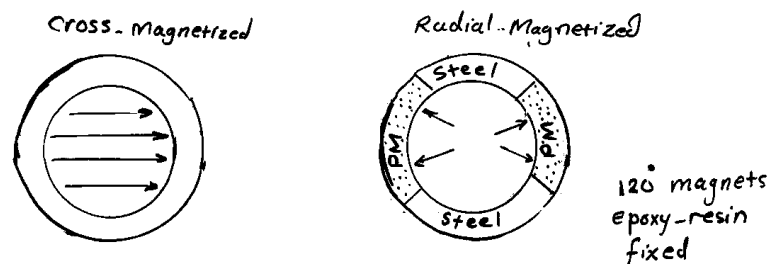


Fig.1: Small PM stators

- The magnet fits within a steel housing which provides the yoke. The magnet manufactured from magnetic materials such as cobalt steel, Tungsten steel or Al, Ni Cobalt alloys ...etc.
- The basic operating equations for PMDC machine are the same as for DC motors or generators. For example, a motor circuit is as shown in Fig. 2. and field winding connections are no longer required.
- As a d.c. motor, the PMDCM equations are:

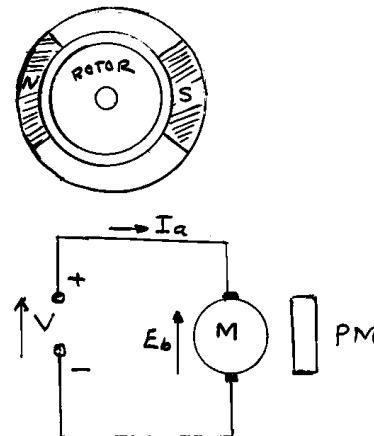


Fig.2
PM DC MOTOR

$$V = E_b + I_a R_a$$

(Neglecting brush voltage drop ΔV_b).

$$E_b = K_e n \Phi = K'_e n$$

Since Φ is constant (PM).

$$\& K'_e = K_e \Phi.$$

$$\therefore V = K'_e n + I_a R_a$$

$$n = \frac{V - I_a R_a}{K'_e \Phi} = \frac{V - I_a R_a}{K'_e} \dots (1)$$

$$T_m = K_T \Phi I_a = K'_T I_a \dots (2)$$

● Speed-Torque characteristics (Mechanical ch/s).

To obtain a relation between the speed and torque of a PM DC motor, we substitute eq.(2) in eq.(1) yields:

$$n = \frac{V - \left(\frac{T_m}{K'_T}\right) R_a}{K'_e} = \frac{V}{K'_e} - \left(\frac{R_a}{K'_T K'_e}\right) T_m \dots (3)$$

At no load $T_m = 0$

$$\therefore n = n_0 = \frac{V}{K'_e}$$

At starting ($n=0$)

$$0 = \frac{V}{K'_e} - \frac{R_a}{K'_T K'_e} T_{mst}$$

$$\therefore T_{mst} = \frac{V}{R_a K'_T}$$

In general $T_{mst} \cong 6 \times T_{FL}$

Equation (3) can be re-written as:

$$n = n_0 - \frac{T_m}{\beta} \dots (4)$$

where $\beta = \frac{K'_T K'_e}{R_a}$ = Coefficient of hardness

β represents the inverse of the slope ($\frac{1}{m}$) of the linear mechanical characteristic of the motor. The

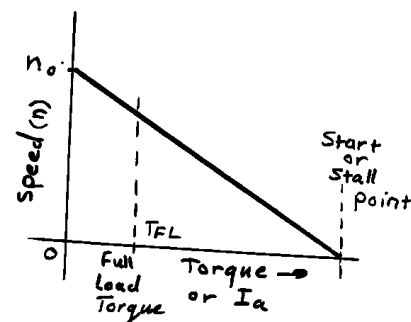


Fig.3
Speed/Torque ch/s.

mechanical ch/s is hard if its slop is small, (this means that a change in the load torque causes small change in the speed), i.e β is large. Due to this reason β is called the coefficient of hardness.

- Effect of Varying V on β :

If the supply voltage V is varied then n_0 in eq.(4) will vary, since $n_0 = \frac{V}{K_e'}$. Hence the characteristic will be shifted as shown in Fig.4 i.e the slop or β will remain unchanged.

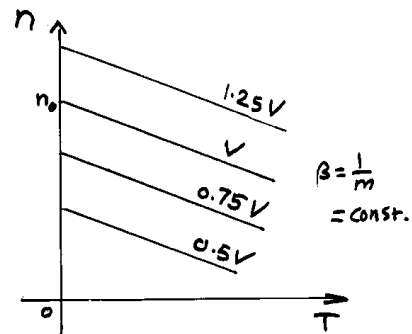


Fig.4.

However n_0 will change as shown in Fig.4, therefore we can control the speed of the motor by changing the applied voltage V .

- Effect of Varying R_a on β :

When we change the value of R_a by inserting an additional resistance R_{add} in the armature circuit as shown in Fig.5, the result is:

$$R_a \Rightarrow \Sigma R_a \equiv R_a + R_{add}.$$

Therefore eq.(1) becomes

$$n = \frac{V - I_a \Sigma R}{K_e'}$$

$$\beta = \frac{K_T K_e'}{\Sigma R_a}$$

&

$$n = n_0 - \frac{T_m}{\beta}$$

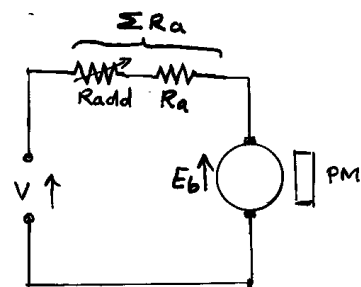
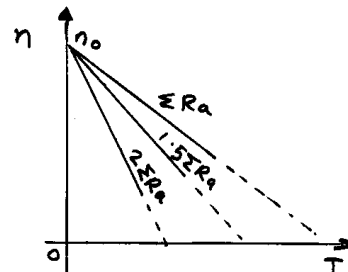


Fig.5

Hence, if we vary ΣR_a , β will be vary as well, but n_0 will remain unchanged and the mechanical ch/s will be as shown in Fig. 6.

- Therefore the speed of the motor n cannot be controlled by varying ΣR_a . only, the coefficient of hardness is affected.



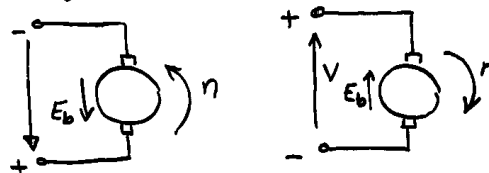
- PM DC Motor has the following advantages compared with field wound DC conventional machines:

1. Higher efficiency : Because it has no losses in the field winding circuit.
2. Less size : No spaces are required for the field windings.
3. Less cost : Small size leads to less cost.

Disadvantages :

1. There is a risk of demagnetization due to excessive armature current or excessive heating.
2. There is a difficulty in operating the motor beyond the base speed. (higher speed than n_0 is difficult to obtain). This is because Φ is constant.

N.B The direction of rotation of a PM DC motor can be reversed only by reversing the polarity of the supply Voltage (V).



PMDC MACHINES

Example 1: A 120V, 1000W PM DC motor draws a current of 4A at no load. When its speed is 1000 rpm, the back e.m.f generated is 110V. The machine constant K_e is unknown. The armature resistance $R_a = 2.5 \Omega$. The permanent magnet produces a flux per pole $\Phi_p = 2.5 \text{ Wb}$. Determine:

- The torque/current characteristic
- The speed / torque characteristic
- The coefficient of hardness of the motor.

Solution :

$$E_b = V - I_a R_a$$

$$= 120 - 4 \times 2.5 = 110 \text{ V}$$

At 1000 r.p.m

$$E_{b_{1000}} = K_e n \Phi$$

$$\therefore 110 = K_e \Phi \times 1000 \times 2.5$$

$$K_e = 0.05 \text{ V/r.p.m/Wb}$$

Now $P_e = P_m$

$$P_e = I_a E_b = K_e n \Phi I_a$$

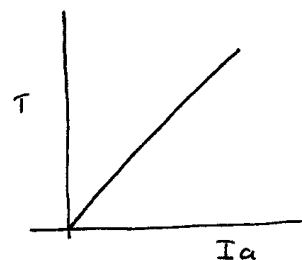
$$P_m = T \cdot \omega = K_T \Phi I_a \cdot \frac{2\pi n}{60}$$

$$K_e n \Phi I_a = K_T \Phi \cdot \frac{2\pi n}{60} I_a$$

$$\therefore K_T = \frac{60}{2\pi} K_e = 9.55 K_e = 9.55 \times 0.05 = 0.4775$$

$$\text{Hence } T = K_T \Phi I_a = 0.4775 \times 2.5 I_a$$

$$\text{or } \boxed{T = 1.19375 I_a}$$



$$K_e' = K_e \phi = 0.125$$

$$K_T' = K_T \phi = 1.935$$

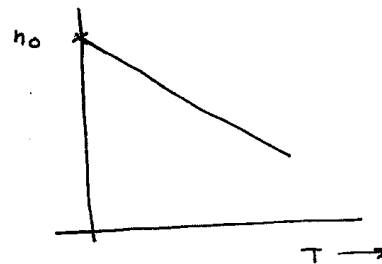
$$n = \frac{V}{K_e'} - m T$$

$$\text{Where } m = \frac{\Sigma R_a}{K_e' K_i'} = \frac{2.5}{0.11 \times 1.935} = 10.34$$

$$n = \frac{120}{0.125} - 10.34 T$$

$$n = 960 - 10.34 T$$

$$n_0 = 960$$



$$c) \text{ Coefficient of hardness } = \beta = \frac{1}{m} = 0.096.$$

Remarks: • Ratings of PM DC motor may be range from 1 W to few kW. They are particularly advantageous in capacities below about 5 hp (1 hp = 746 W). The magnets are ceramic or rare-earth/cobalt alloys.

- Elongated armature in PM motors ensure low inertia and fast response. So they are mainly used in SERVO applications.

- Typical 1.5 hp PM DC motor has the following specification:

1.5 hp, 90 V, 2900 r.p.m. 14.5 A

Armature length = 115 mm

No. of slots = 20

No. of Commutator segments = 40

turns per coil = 5

Conductor size : No. 17 AWG (American wire gage).

Armature resistance = 0.34 Ω at 20°C.

Example :- Given a permanent magnet d.c. motor whose armature resistance is 0.8Ω and that operates on 24V supply. Assume the air-gap flux is 0.002 Wb , and the motor torque constant $K_T = 100 \text{ N.m/Wb}$. If the motor is operating under a load torque of 1 N.m , calculate:

- the speed of the motor in rpm and rad/sec.
- the Stalling torque of the motor.

Solution:-

$$(a) \quad n = \frac{V_t}{K_e \phi} - \frac{R_a}{K_T K_e \phi^2} T.$$

$$K_e = \frac{K_T}{9.55} = \frac{100}{9.55} = 10.47$$

$$n = \frac{24}{(10.47)(0.002)} - \frac{0.8 \times 1}{100 \times 10.47 (0.002)^2}$$

$$= 1146.1318 - 191 = 955 \text{ r.p.m.}$$

$$\omega = \frac{2\pi}{60} \times n = \frac{2\pi}{60} \times 955 = 100 \text{ rad/sec.}$$

(b) The Stalling torque T_{stall} can be found by setting $n = 0$.

$$0 = \frac{V_t}{K_e \phi} - \frac{R_a}{K_T K_e \phi^2} T_{\text{stall}}$$

$$\frac{V_t}{K_e \phi} = \frac{R_a}{K_T K_e \phi^2} T_{\text{stall}}$$

$$\therefore T_{\text{stall}} = \frac{K_T K_e \phi^2 \cancel{V_t}}{K_e \phi R_a} = \frac{K_T \phi V_t}{R_a}$$

$$= \frac{100 \times 0.002 \times 24}{0.8} = \underline{\underline{6 \text{ N.m}}}$$

Ans.

PMDC MOTORS

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Example 3: A 200W small PMDC motor running at 1400 r.p.m when connected to 48V d.c. supply. The armature resistance of the motor is 1.5Ω , and the machine constant $K_e = 7.87$ V/rpm. If the rotational losses of the motor $P_{rot} = 40$ W, and the motor is delivering its rated output, calculate the flux per pole.

Solution

$$P_d = P_{out} + P_{rot} = 200 + 40 = 240 \text{ W}.$$

$$\omega = \frac{2\pi}{60} \times n = \frac{2\pi}{60} \times 1400 = 147 \text{ rad/s}.$$

$$\therefore T_d = \frac{P_d}{\omega} = \frac{240}{147} = 1.63 \text{ N.m}$$

$$K_T = K_e \times 9.55 = 7.87 \times 9.55 = 75.206$$

$$n = \frac{V_t}{K_e \phi} - \frac{R_a}{K_e K_T \phi^2} T_d \Rightarrow \times \frac{\phi^2}{n}$$

$$\phi^2 = \frac{V_t}{K_e n} \phi - \frac{R_a}{K_e K_T n} T_d$$

$$\phi^2 - \frac{V_t}{K_e n} \phi + \frac{R_a}{K_e K_T n} T_d = 0.$$

$$\phi^2 - \frac{48}{7.87 \times 1400} \phi + \frac{1.5 \times 1.63}{7.87 \times 75.206 \times 1400}$$

$$\phi^2 - 0.004 \phi + 2.9 \times 10^{-6} = 0$$

$$\phi = \frac{+0.004 \pm \sqrt{(0.004)^2 - 4 \times 1 \times 2.9 \times 10^{-6}}}{2 \times 1}$$

$$\therefore \phi = 3.04 \times 10^{-3} \text{ Wb}.$$

N.B: $Ax^2 + Bx + C = 0$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Example 2:

Permanent Magnet DC Motor (PMDC) . Speed Control.

A 100V , 1120W (1.5hp) , 2900 r.p.m , Permanent magnet dc motor has an armature resistance of 0.5Ω , is taken armature current of 14.0A at rated load. Calculate the value of the external resistance placed in series with the armatur circuit to reduce the speed of the motor to 1800 r.p.m . Assuming the torque to be constant and neglect brush voltage drop .

Solution:

As the torque remains constant, the armature current I_a will remain constant since $T \propto I_a$.
(The flux in a PM motor is also constant).

$$n_1 = \frac{V - I_{a1} \Sigma R_{a1}}{k \phi} \quad \dots (1)$$

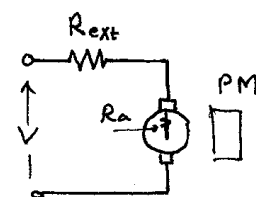
$$n_2 = \frac{V - I_{a2} \Sigma R_{a2}}{k \phi} \quad \dots (2)$$

Deviding (2) by (1) we get :

$$\frac{n_2}{n_1} = \frac{V - I_{a2} \Sigma R_{a2}}{V - I_{a1} \Sigma R_{a1}} = \frac{V - I_a \Sigma R_{a2}}{V - I_a \Sigma R_{a1}} \quad (\text{since } I_{a1} = I_{a2} = I_a)$$

$$\frac{1800}{2900} = \left[\frac{100 - (14.0) \Sigma R_{a2}}{100 - (14.0)(0.5)} \right]$$

$$(100 - 7) = 161 - 22.54 \Sigma R_a$$



$$\therefore \Sigma R_{a2} = \frac{161 - 93}{22.54} = 3.016 \Omega$$

but $\Sigma R_{a2} = R_{ext} + R_a$

$$\therefore R_{ext} = \Sigma R_{a2} - R_a = 3.016 - 0.5 = \underline{\underline{2.5 \Omega}}$$

Small Electrical Machines

Dr. M.T.Lazim

Synchronous Machines

1. Synchronous Generator :-

Synchronous machines are very important type of electrical machines. Electrical power is generated almost entirely by synchronous generators, of individual rating up to 1GW. (one gega watts) .

Like dc machines, a synchronous machine can also operate as both a generator and a motor. The synchronous generators are built with either stationary or rotating DC magnetic field.

- A stationary - field synchronous generator has the same shape as d.c. generator. The salient poles create the d.c. field. The rotor (armature) carries 3-phase winding whose terminals connected to 3 slip rings mounted on the machine rotor shaft.
- A revolving - field synchronous generator has a stationary armature called "STATOR". The stator also accomodates 3-phase windings that are directly connected to the load. The field winding is mounted on the rotor as shown in Fig.1. The rotor is either salient-pole type or nonsalient (cylindrical) type

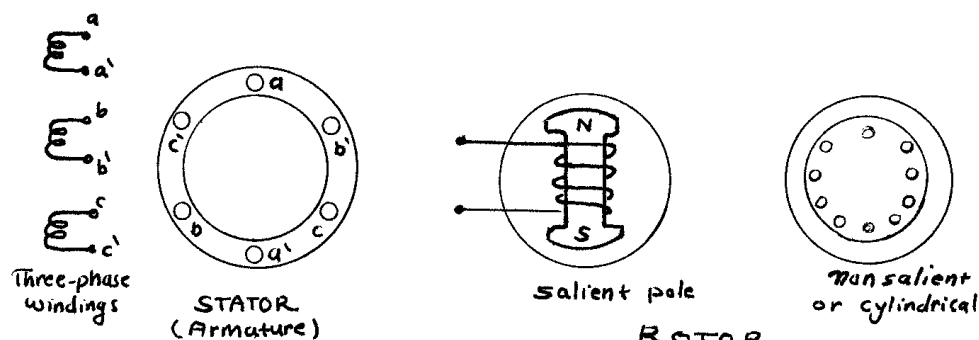
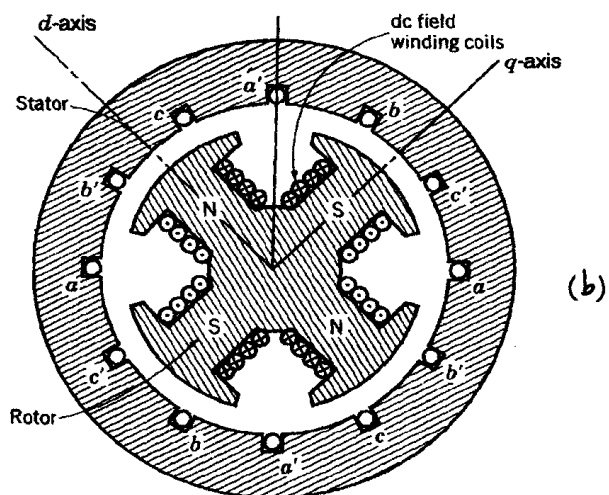
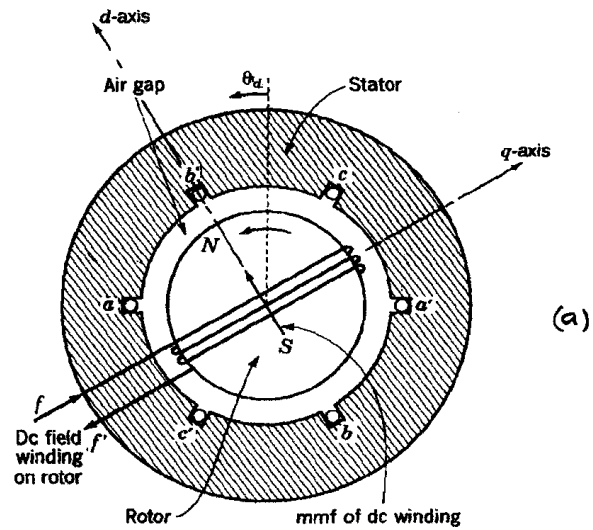


Fig.1



Synchronous machine types

(a) Cylindrical rotor type. (b) Salient pole type

For a revolving field synchronous generator, the stator winding A has conductors which are successively swept by the N and S poles of the rotor. If a positive voltage is induced when an N pole sweeps across the conductors, a similar negative voltage is induced when the S pole speed by. Thus every time a complete two-poles crosses the conductors, the induced voltage goes through a complete cycle of period T.

Let:

period of one cycle = T

$$\therefore f = \text{frequency of the induced voltage} \\ = \frac{1}{T}$$

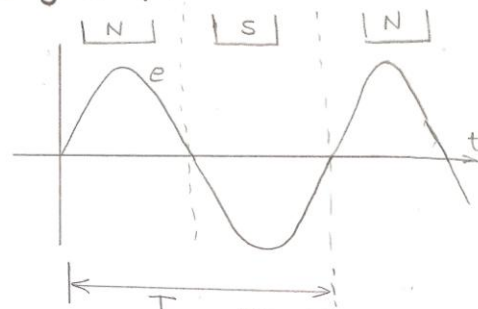


Fig. 2A

If we have p poles on the machine, then the frequency of the induced voltage will be :

$$f = \frac{P}{2} \times \frac{n_s}{60} = \frac{P n_s}{120} \quad \text{Hz} \cdot (\text{cycle/sec})$$

where n_s = speed of the rotor in r.p.m. (synchronous speed)

Example: If a synchronous generator runs at 200 rpm and the frequency of the induced voltage is 50 Hz, how many poles does the rotor have?

Solution :

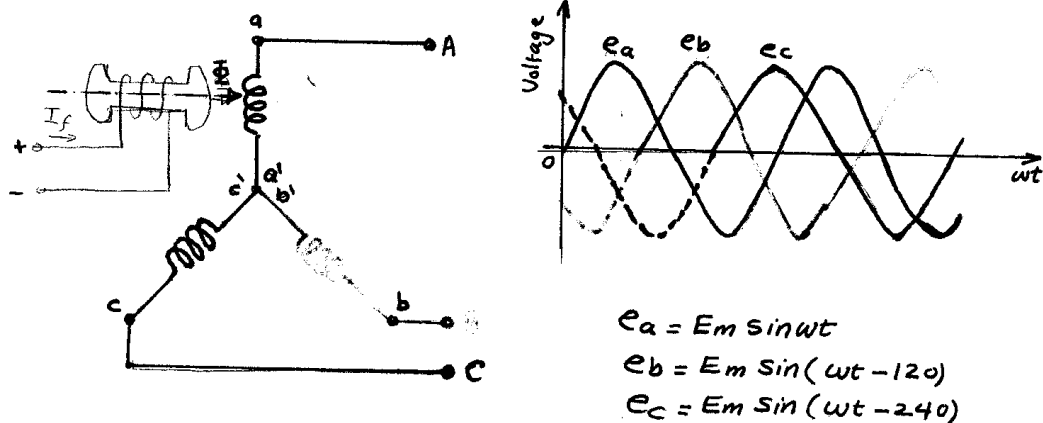
$$f = \frac{P n_s}{120}$$

$$50 = \frac{P \times 200}{120}$$

$$\therefore p = \frac{50 \times 120}{200} = 30 \text{ poles} \\ \text{or 15 pairs of N and S poles.}$$

1.1 The 3-phase Voltages of a Synchronous Generator

The field carries an exciting current " I_f " which produces a flux Φ . As the field revolves, the flux induces in the stator three equal voltages E that are 120° out of phase are generated.



Induced Voltages:

Let Φ_p = flux per pole

$$\lambda(wt) = \text{flux linkage} = N \Phi_p$$

N = Number of turns per winding.

For winding a , the flux linkage for winding $a'a'$ will be maximum ($= N \Phi_p$) at $wt = 0$ (Fig. 3) and zero at $wt = 90^\circ$.

The flux linkage $\lambda_a(wt)$ will vary as the cosine of the angle wt . Hence $\lambda_a(wt) = N \Phi_p \cos wt$

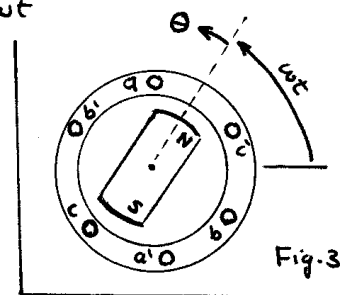
$$e_a = -\frac{d\lambda_a(wt)}{dt} = \frac{d}{dt}[N \Phi_p \cos wt]$$

$$e_a = \omega N \Phi_p \sin wt$$

$$= E_m \sin wt$$

where $E_m = \omega N \Phi_p = 2\pi f \Phi_p N$

& $E_{r.m.s} = \frac{2\pi f}{\sqrt{2}} N \Phi_p = 4.44 f N \Phi_p \dots\dots (1)$



The voltages induced in the other phase coils are also sinusoidal, but phase-shifted from each other by 120 electrical degrees. Thus,

$$e_b = E_m \sin(\omega t - 120)$$

$$e_c = E_m \sin(\omega t - 240)$$

However, e_a, e_b and e_c have same r.m.s values
i.e. $E_{a,r.m.s} = E_{b,r.m.s} = E_{c,r.m.s} = \frac{E_m}{\sqrt{2}} = E_o$.

1.2 Equivalent Circuit Model

Since the three voltages generated in synchronous machine are identical in waveform and have same r.m.s value E_o , therefore representation of one phase is sufficient to model the machine. The equivalent circuit per phase of a synchronous machine is shown in Fig.4

E_o = r.m.s induced voltage
per phase

X_s = reactance of the
armature coil (aa')
per phase

R_a = armature resistance
per phase

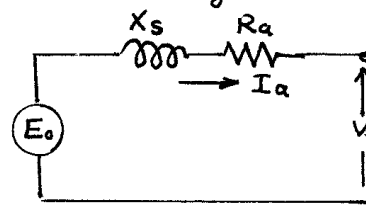


Fig.4. Equivalent circuit per phase

For the three-phases, the circuit of the synchronous machine will be as shown in Fig.5

$$Z_s = \sqrt{R_a^2 + X_s^2} = \text{Synchronous Impedance}$$

V_t = terminal voltage
of the generator

Hence:

$$E_o = V_t + I_a R_a + j I_a X_s$$

or $E_o = |E_o| \angle \delta$

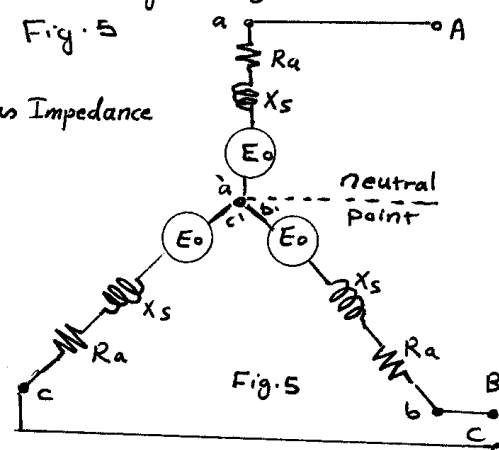


Fig.5

1.3 Phasor Diagram

The phasor diagram of a synchronous generator can be obtained by taking the terminal voltage V_t as the reference phasor in constructing the phasor diagram. Hence, for the per-phase equivalent circuit of the synchronous generator, the phasor diagram is

$$V_t = |V_t| \angle 0^\circ$$

$$E_o = |E_o| \angle \delta$$

$$I_a = |I_a| \angle -\phi$$

ϕ : is the synchronous impedance

$$\text{angle } Z_s = \sqrt{R_a^2 + X_s^2} = |Z_s| \angle \phi$$

$$\text{where } \phi = \tan^{-1} \frac{X_s}{R_a}.$$

δ : is the angle between E_o and V_t and called the load angle or power angle.

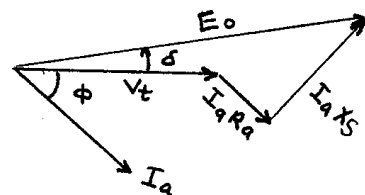


Fig.7
Phasor
Diagram

Example.1 : A 3-phase, 5kVA, 208V, four-pole, 50 Hz, star-connected synchronous generator has stator winding resistance per phase $R_a = 0.2 \Omega$ and a synchronous reactance per phase $X_s = 8 \Omega$. Determine the generated voltage E_o and the power angle when the generator delivers rated KVA at 0.8 power factor lagging. Draw the phasor diagram for this condition.

Solution :

The per-phase equivalent circuit of the generator is shown in Fig.8(a):

For star connection

$$V_t = \frac{208}{\sqrt{3}} = 120 \text{ V/phase}$$

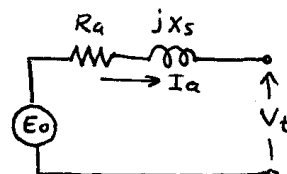


Fig.8(a)

Stator current at rated KVA :

$$\text{KVA/phase} = \frac{5000}{3}$$

$$\therefore I_a = \frac{5000}{3 V_t} = \frac{5000}{3 \times 120} = 13.9 \text{ A} .$$

$$(\text{since } \text{KVA} = 3 I_a V_t)$$

$$\text{P.f} = \cos \phi = 0.8$$

$$\therefore \phi = \cos^{-1} 0.8 = 36.9^\circ .$$

$$\text{for lagging P.f } \phi = -36.9^\circ .$$

$$\begin{aligned} E_o &= V_t \angle 0^\circ + I_a R_a + I_a j X_s \\ &= 120 \angle 0^\circ + 13.9 \times 0.2 \angle -36.9^\circ + 13.9 \angle -36.9^\circ \times 8 \angle +90^\circ \\ &\approx 210 \angle 26^\circ \end{aligned}$$

$$\therefore E_o = 210 \text{ V/phase}$$

$$\text{power angle} = +26^\circ .$$

The phasor diagram is shown in Fig. 8(b) .

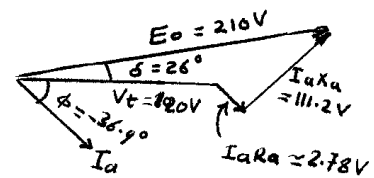


Fig8(b): phasor diagram

Synchronous Motor

The synchronous motor has same construction as Synchronous generator, except that its terminals of the stator is connected to the A.C. Supply (3 ϕ supply), as shown in Fig.10

The synchronous speed of the motor is given by

$$n_s = \frac{120f}{p}$$

where: f = frequency of the supply (Hz)

p = number of poles

n_s = synchronous speed.

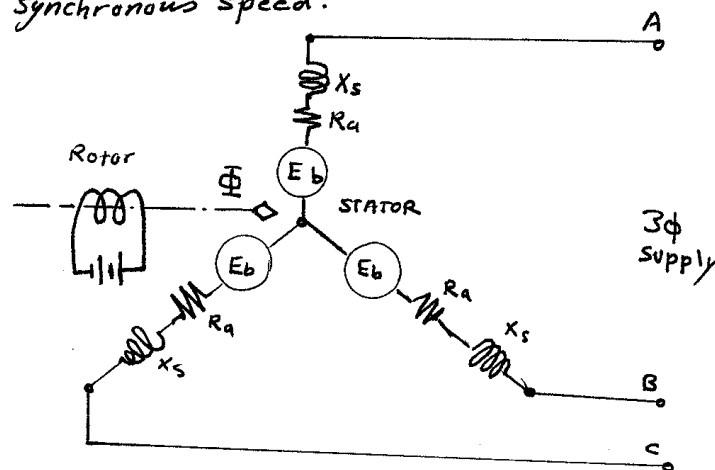
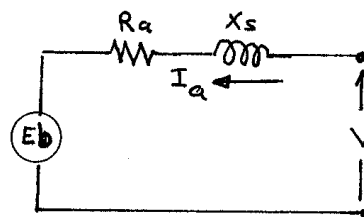


Fig.10. Synchronous motor connected to 3-phase supply.

Equivalent Circuit:

Fig.11 shows the equivalent circuit of a synch. motor for one phase.



$$V = \text{Supply voltage per phase}$$

$$= \frac{V_{\text{line}}}{\sqrt{3}}$$

Fig.11. Equivalent circuit per phase of a synchronous motor. (Exact CCT).

E_b = generated back e.m.f of the motor per phase.

Phasor Diagram & power

Consider a synchronous motor connected to an a.c. voltage source V , as shown in Fig. 12.

Assume that $X_s \gg R_a$ (we can neglect R_a).

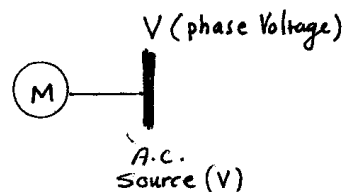
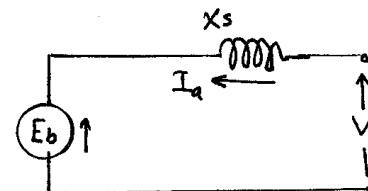


Fig. 12.



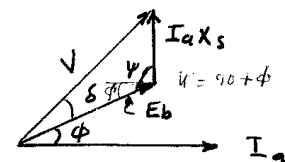
Approx. Equivalent CCT/phase

$$V = E_b + jI_a X_s$$

If I_a lags E_b by angle ϕ
and V leads E_b by δ

The active power absorbed by the motor :

$$P_e = E_b I_a \cos \phi$$



Phasor diagram
Per phase

From the sine law for triangle s : $\frac{V}{\sin \psi} = \frac{I_a X_s}{\sin \delta}$

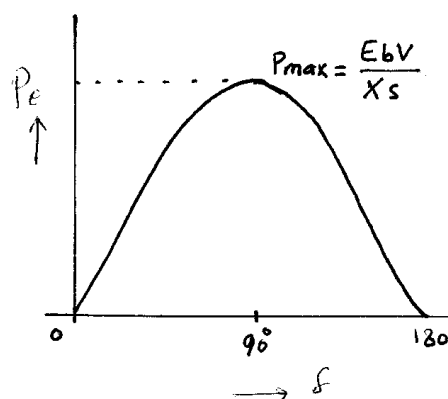
but $\psi = 90 + \phi$ $\therefore \sin \psi = \sin(90 + \phi) = \cos \phi$

Hence $\frac{V}{\cos \phi} = \frac{I_a X_s}{\sin \delta}$ or $I_a \cos \phi = \frac{V}{X_s} \sin \delta$

$$\text{or } P_e = \frac{E_b V}{X_s} \sin \delta$$

P_e is maximum
when $\delta = 90^\circ$.

$$\therefore P_{\max} = \frac{E_b V}{X_s}$$



Power & Torque

When the motor operates under load, it draws active power from the a.c. supply line. If we neglect R_a ($R_a \approx 0$). Then

$$P_e = P_m = \frac{V \cdot E_b}{X_s} \sin \delta \quad (\text{neglecting rotational losses}).$$

Where P_m is the mechanical power

$$\text{When } \delta = 90^\circ, P_m = P_{m \max} = \frac{V \cdot E_b}{X_s}.$$

Note: V is the phase voltage.

As far as the torque is concerned, it is directly proportional to the mechanical power:

$$P_m = T \omega = T \cdot \frac{2\pi n_s}{60} \quad n_s = \text{synch. speed.}$$

$$\text{or } \boxed{T = \frac{30}{\pi} \frac{P_m}{n_s}}.$$

Pull-out torque: when the motor is loaded by its maximum power $\delta = 90^\circ$ the motor will lose its synchronism. and the maximum torque occurs at maximum power loading is called pull-out torque.

Example: A 150 kW, 1200 r.p.m 460V, 3-phase synchronous motor has a synchronous reactance of 0.8Ω , per phase. If the excitation (back e.m.f) voltage E_b is fixed at 300V, per phase. Determine the following:

- The power vs δ curve.
- The torque vs δ curve.
- The pull-out torque of the motor.

Solution :

(a) The line-to-neutral Voltage is

$$V = \frac{V_L}{\sqrt{3}} = \frac{460}{\sqrt{3}} = 266 \text{ V.}$$

The mechanical power per phase:

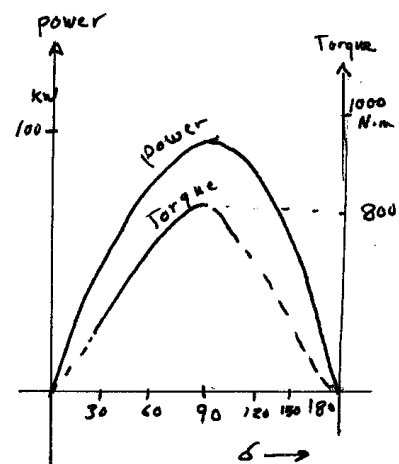
$$\begin{aligned} P &= \frac{E_o V}{X_s} \sin \delta \\ &= \frac{266 \times 340}{0.8} \sin \delta \\ &= 99750 \sin \delta \text{ W} \\ &\approx 100 \sin \delta \text{ KW} \end{aligned}$$

$$(b) T = 9.55 \frac{P}{1200} = \frac{P}{125}$$

(c) The pull-out torque T_{max} coincide with the maximum power output:

$$T_{max} = 800 \text{ N.m.}$$

$\delta [^\circ]$	$P [\text{KW}]$	$T [\text{N.m}]$
0	0	0
30	50	400
60	86.6	693
90	100	800
120	86.6	693
150	50	400
180	0	0



Small Electrical Machines	Dr. Mohammed Tawfeeq
Tutorial Sheet 4	

1. A 380-V, 50 Hz, four-pole small synchronous motor draws 5 A from the line at unity power factor and full load. Assuming that the motor is lossless, answer the following questions:

- (a) What is the output torque of this motor?
- (b) What must be done to change the power factor to 0.8 leading? Explain your answer, using phasor diagrams.
- (c) What will the magnitude of the line current be if the power factor is adjusted to 0.8 leading?

[Ans: (a) 17.45 N.m, (c) 6.25A]

2. A 380-V, 50 Hz, 3000 W, 0.8-PF-leading six-pole Δ -connected small synchronous motor has a synchronous reactance of 11Ω and negligible armature resistance. Ignore its friction, windage, and core losses for the purposes of this problem.

- (a) If this motor is initially supplying 3000W at 0.8 PF lagging, what are the magnitudes and angles of E_o and I_a ?
- (b) How much torque is this motor producing? What is the torque angle δ ? How near is this value to the maximum possible induced torque of the motor for this field current setting?
- (c) If E_o is increased by 15 percent, what is the new magnitude of the armature current? What is the motor's new power factor?

[Ans: (a) $359.56 \angle -4.5^\circ$ V, $3.28 \angle -36.87^\circ$ A, (b) 28.65 N.m, (c) $\delta = -3.9^\circ$]

3. A 400-V three-phase Y-connected small synchronous motor has a synchronous reactance of 8Ω per phase. The field current has been adjusted so that the torque angle δ is 18° when the power supplied by to the motor is 5 kW.

- (a) What is the magnitude of the internal generated voltage E_o in this machine?
- (b) What are the magnitude and angle of the armature current in the machine? What is the motor's power factor?
- (c) If the field current remains constant, what is the absolute maximum power this motor could supply?

[Ans: (a) 186.85V, (b) $9.74 \angle 42.2^\circ$ A, (c) 16.11 kW]

4. A 1 hp, 2 - pole small reluctance motor operating on 120V, 50 Hz supply under rated load condition. The motor torque angle is 25° . Determine:

- (a) The load torque on the motor shaft,
- (b) The torque angle δ if the supply voltage dropped to 110V.
- (c) Will the motor pull-out of synchronism at this torque angle?

[Ans: (a) 2.3 N.m, (b) $\delta = 32.86^\circ$]

5. A brushless DC motor is operating from an ideal battery of voltage $V_B = 120$ V. The motor phase winding resistance is 0.4 ohms. At a speed of 3000 rpm with the motor open circuit, the magnitude (2em) of the trapezoidal line-to-line voltage was determined to be 73.34 V. If the BLCD is operating at 4500 rpm,
- (a) Sketch a compatible stator voltage and current waveforms indicating the conduction sequence of the controller switching devices;
 - (b) Find the magnitude of phase currents and total developed torque. Assume that the forward voltage drop of a conducting transistor is 0.7 V. The induced voltage of the two series phase coils is determined by direct ratio of speed.
 - (c) Calculate the motor efficiency if the core loss $P_{cl} = 70$ W and the stray losses $P_s = 95$ W at 4500 rpm.

[Ans : 10.75 A, 2.5 N.m]

Small Electrical Machines
Problem sheet No. 4 problem solution.

Problem-1

(a) Assuming the motor is lossless

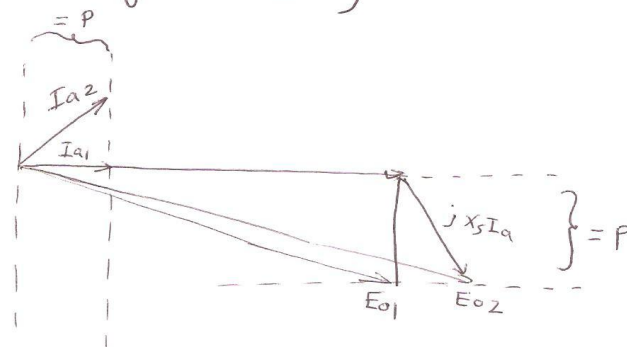
$$P_{in} = P_{out} = \sqrt{3} V_t I_L \cos \theta$$

$$= \sqrt{3} \times 380 \times 5 \times (1.0) = 3290.8 \text{ W}$$

The output torque

$$T_L = \frac{P_{out}}{\omega_m} = \frac{3290.8}{\frac{2\pi \times 1800}{60}} = 17.45 \text{ N.m.}$$

(b) To change the motor's power factor to 0.8 leading, its field current must be increased. Since the power supplied to the load is independent of the field current level, an increase in the field current increases $|E_o|$ while keeping the distance $E_o \sin \delta$ constant. This increase in E_o changes the angle of the current I_a , eventually causing it to reach a power factor of 0.8 leading



$$Q = I_a \sin \theta$$

(c) The magnitude of the line current will be:

$$I_L = I_a = \frac{P}{\sqrt{3} V_t \text{PF}} = \frac{3290.8}{\sqrt{3} \times 380 \times 0.8} = 6.25 \text{ A}$$

problem 2

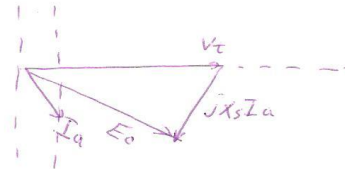
$$(a) I_L = \frac{3000}{\sqrt{3} V_t \text{PF}} = \frac{3000}{\sqrt{3} \times 380 \times 0.8} = 5.69 \text{ A}$$

since the machine is Δ -connected, thus

$$I_a = \frac{5.69}{\sqrt{3}} = 3.28 \text{ A}$$

$$-\cos^{-1}(0.8) = -36.87^\circ$$

$$\text{So } I_a = 3.28 \angle -36.87^\circ \text{ A}$$



$$\begin{aligned} (b) E_o &= V_t - jX_s I_a \\ &= 380 \angle 0^\circ - j 11 \times 3.28 \angle -36.87^\circ \\ &= 380 - j (11)(3.28) \{ \cos(-36.87) + j \sin(-36.87) \} \\ &= 380 - j 28.8 + 21.6 = 401.6 - j 28.8 \\ &= 402 \angle -4.5^\circ \end{aligned}$$

$$(b) n_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

$$\omega_m = \frac{2\pi}{60} n_s = \frac{1000}{9.55} = 104.7 \text{ rad/sec.}$$

$$\therefore T_{out} = \frac{P_{out}}{\omega_m} = \frac{3000}{104.7} = 28.65 \text{ N.m.}$$

(C) If E_o increased by 15%, the new torque angle can be found from the fact that:

$$E_o \sin \delta \propto P = \text{constant}$$

$$E_{o2} = 1.15 E_{o1} = 1.15 \times 402 = 462.3 \text{ V.}$$

$$\begin{aligned} \delta_2 &= \sin^{-1} \frac{E_{o1} \sin \delta_1}{E_{o2}} \\ &= \sin^{-1} \frac{402 \sin(-4.5^\circ)}{462.3} = -3.9^\circ \\ &= \sin^{-1}(0.869)(-0.078) = -3.9^\circ \end{aligned}$$

Problem-3

(a) The power supplied to the motor is 5 kW. This power is given by the equation:

$$P = \frac{3V_{ph} E_o}{X_s} \sin \delta$$

$$V_{ph} = \frac{V_{tL}}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230. V.$$

$$\therefore E_o = \frac{X_s P}{3V_{ph} \sin \delta} = \frac{8 \times 5000}{3 \times (230) \sin 18} = 186.85 V.$$

(b) The armature current in this machine is

$$\begin{aligned} I_a &= \frac{V_{ph} - E_o}{jX_s} = \frac{230 \angle 0^\circ - 186.85 \angle -18^\circ}{j8} \\ &= -j28.75 + j23.35 \angle -18^\circ \\ &= j \{-28.75 + 23.35 \{\cos 18 - j \sin 18\}\} \\ &= j \{-28.75 + 22.2 - j7.21\} = j\{-6.55 - j7.21\} \\ &= 7.21 - j6.55 = \sqrt{42.9 + 51.28} = 9.74 A \angle 42.2^\circ. \end{aligned}$$

(c) The maximum power that this motor could supply:

$$\begin{aligned} P_{max} &= \frac{3V_{ph} E_o}{X_s} \sin 90 = \frac{3 \times 230 \times 186.85}{8} \\ &= 16.116 \text{ kW}. \end{aligned}$$

problem -4

A 750 W 2-pole, 120 V, 50 Hz small reluctance motor, operating under rated load condition. The motor torque angle is 45° . Determine:

- Torque load on the shaft (average torque).
- Torque angle if the supply voltage dropped to 100 V.
- Will the rotor pull-out of synchronism?

Solution

$$(a) \quad n = \frac{120 \times f}{P} = \frac{120 \times 50}{2} = 3000 \text{ rpm.}$$

$$P_{\text{shaft}} = T_{\text{av}} \cdot \omega_s = \frac{750 \times 60}{2\pi \times 3000} = \frac{9.55}{4} = 2.38 \text{ N.m.}$$

$$(b) \quad T_{\text{av}1} = T_{\text{av}2}.$$

$$V_1^2 \sin 2\delta_1 = V_2^2 \sin 2\delta_2.$$

$$\sin 2\delta_2 = \left(\frac{120}{100}\right)^2 \sin 50^\circ = 1.44 \times 0.766 = 1.103$$

$$= 0.911$$

$$2\delta_2 = \sin^{-1} 0.911$$

$$\therefore 2\delta_2 = 65.7$$

$$\therefore \delta = 32.86.$$

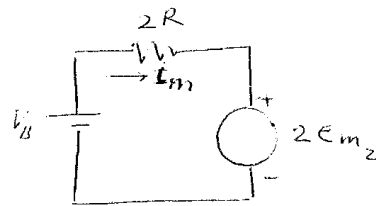
- Because $\delta_2 < 45^\circ$, the rotor will not lose synchronism.

problem 5

(b)

$$I_m = \frac{V_B - 2V_f - 2E_{m2}}{2R}$$

$$= \frac{120 - 1.4 - 2E_{m2}}{2 \times 0.4}$$

To find $2E_{m2}$, we have $e \propto n$

$$\therefore \frac{2E_{m2}}{2E_{m1}} = \frac{n_2}{n_1} = \frac{4500}{3000} = 1.5$$

$$\therefore 2E_{m2} = 73.34 \times 1.5 = 110 \text{ V}$$

$$\therefore I_m = \frac{120 - 1.4 - 110}{0.8} = 10.75 \text{ A}$$

The torque is given by:

$$3T_d = \frac{2E_m I_m}{\omega_m} = \frac{110 \times 10.75}{\frac{2\pi \times 4500}{60}} = \underline{\underline{2.5 \text{ N}\cdot\text{m}}}$$

$$(c) \text{ Output power } P_{out} = 3T_d \omega = 2.5 \times \frac{4500}{9.55} = 1178 \text{ W}$$

Copper losses

$$P_{cu} = (10.75)^2 \times 0.4 \times 3 = 46.2 \times 3 = 138.6 \text{ W}$$

$$P_{cl} = 70 \text{ W}$$

$$P_s = 95 \text{ W}$$

$$\therefore \eta = \frac{P_{out}}{P_{out} + \text{losses}} = \frac{1178}{1178 + 138.6 + 70 + 95} = \frac{1178}{1481.6}$$

$$= 0.795 \quad \text{or } \underline{\underline{79.5\%}}$$

Brushless DC electric motor

A **brushless DC motor (BLDC)** is an AC synchronous motor with a permanent magnet in the rotor that looks very similar to a DC motor.

In a conventional (brushed) DC-motor, the brushes make mechanical contact with a set of electrical contacts on the rotor (called the **commutator**), forming an electrical circuit between the DC electrical source and the armature coil-windings. In a BLDC motor, the electromagnets do not move; instead, the permanent magnets rotate and the armature remains static. In order to do this, the brush-system/commutator assembly is replaced by an intelligent electronic controller. The controller performs the same power-distribution found in a brushed DC-motor, but using a solid-state circuit rather than a commutator/brush system.

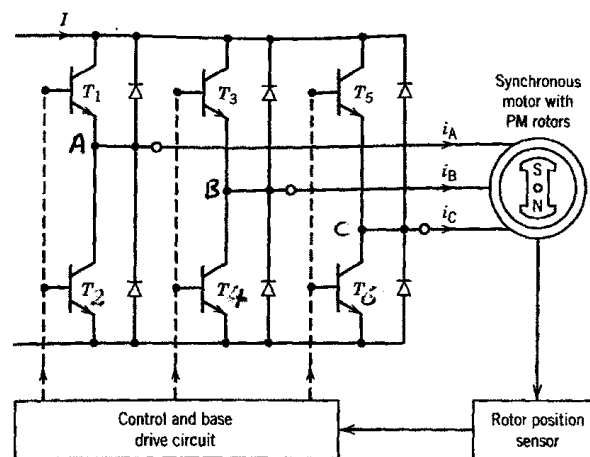


Fig.1

Controller Implementations

Because the controller must direct the rotor rotation, the controller needs some means of determining the rotor's orientation/position (relative to the stator coils.) Some designs use Hall effect sensors or a rotary encoder to directly measure the rotor's position. Like an AC motor, the voltage on the undriven coils is sinusoidal, but over an entire commutation the output appears trapezoidal because of the DC output of the controller.

The controller shown in fig.1 contains 3 bi-directional drivers to drive high-current DC power, which are controlled by a logic circuit ,each driver contains two power transistors, i.e. the controller has six power transistors T1,T2,T3,T4,T5 and T6.. Simple controllers employ comparators to determine when the output phase should be advanced, while more advanced controllers employ a microcontroller to manage acceleration and control the speed.

Now if the controller is acting such that the three transistors T1, T4 and T5 are gated, then terminals A and C have the battery voltage, while terminal B has zero voltage. In this state , a current will flow from terminal A to B , and another current from C to B as illustrated in Fig.2.The solid arrows indicate the direction of the field in each phase due to its current . The big arrow in the center is the resultant magnetic field.

The rotor is placed in such a position that the field flux will have 90° angle with respect to the stator magnetic field as shown in Fig. 2.In such state a clockwise torque will be produced on the rotor. After it revolves about 30° , transistor T5 is turned off and T6 turned on which makes the stator magnetic field revolves 60° clockwise. Thus when the rotor south pole gets near the stator north ,the north goes away further to create the continuous pole(clockwise rotation. The on-off sequence and the rotation of the magnetic fields in the stator are shown in Fig.3.

Motor Construction

BLDC motors can be constructed in different physical configurations: In the 'conventional' configuration, the permanent magnets are mounted on the rotor. Three stator windings surround the rotor. From a controller standpoint, the two styles of windings are treated exactly the same, although some less expensive controllers need to read voltage from the common center of the wye winding.The stator winding connection is shown in Fig4.

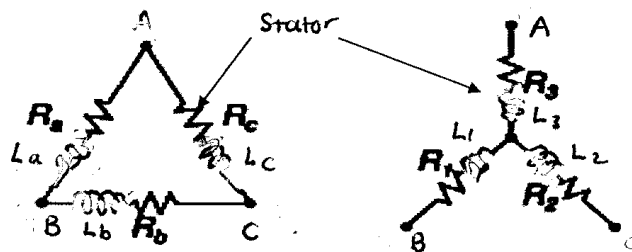


Fig.4 Schematic for delta and wye winding styles for stator windings connection.

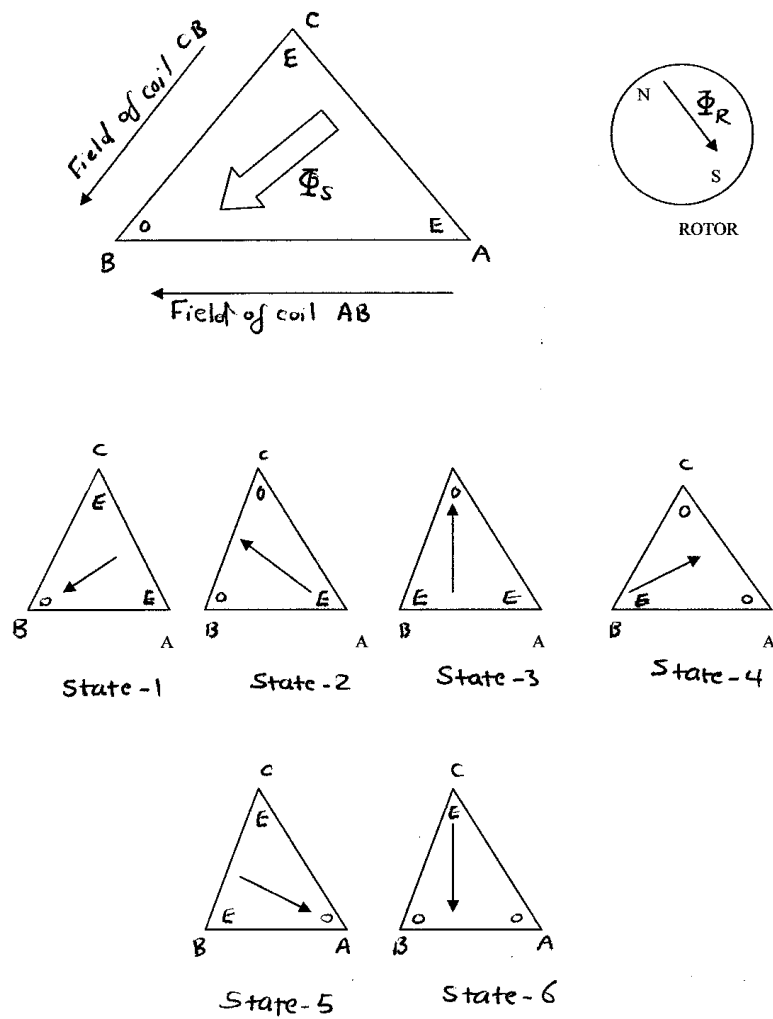


Fig.2. Stator magnetic field direction

Transistor	State 1 Logic	State 2 Logic	State 3 Logic	State 4 Logic	State 5 Logic	State 6 Logic
T1	1	1	1	0	0	0
T2	0	0	0	1	1	1
T3	0	0	1	1	1	0
T4	1	1	0	0	0	1
T5	1	0	0	0	1	1
T6	0	1	1	1	0	0

Fig.3 Logic table

Rotor Construction

Fig.4 shows a simple rotor construction

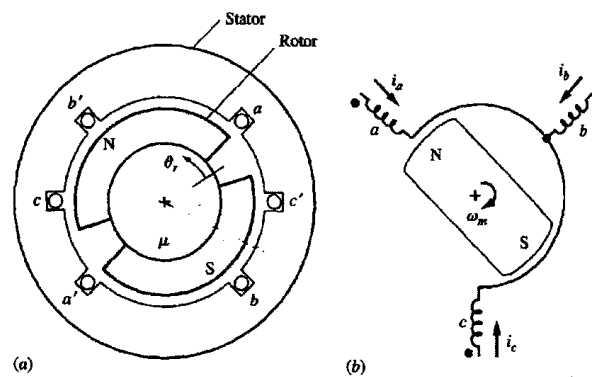


Fig.4. Permanent magnet synchronous machine (a) Physical, (b) Schematic

Advantages and Disadvantages

BLDC motors offer several advantages over brushed DC-motors, including:

1. higher efficiency and reliability,
2. reduced noise,
3. longer lifetime (no brush erosion),
4. elimination of ionizing sparks from the commutator, and
5. overall reduction of electromagnetic interference (EMI).

The maximum power that can be applied to a BLDC motor is exceptionally high, limited almost exclusively by heat, which can damage the magnets.

The main disadvantage is higher cost, which arises from the issue that BLDC motors require complex electronic speed controls to run.

Applications

BLDC motors can potentially be deployed in any field-application currently fulfilled by brushed DC motors. BLDC motors have come to dominate many applications: Consumer devices such as computer hard drives, CD/DVD players, and PC cooling fans use BLDC motors almost exclusively. Low speed, low power brushless DC motors are used in direct-drive turntables.

Power and torque of BLDC motor

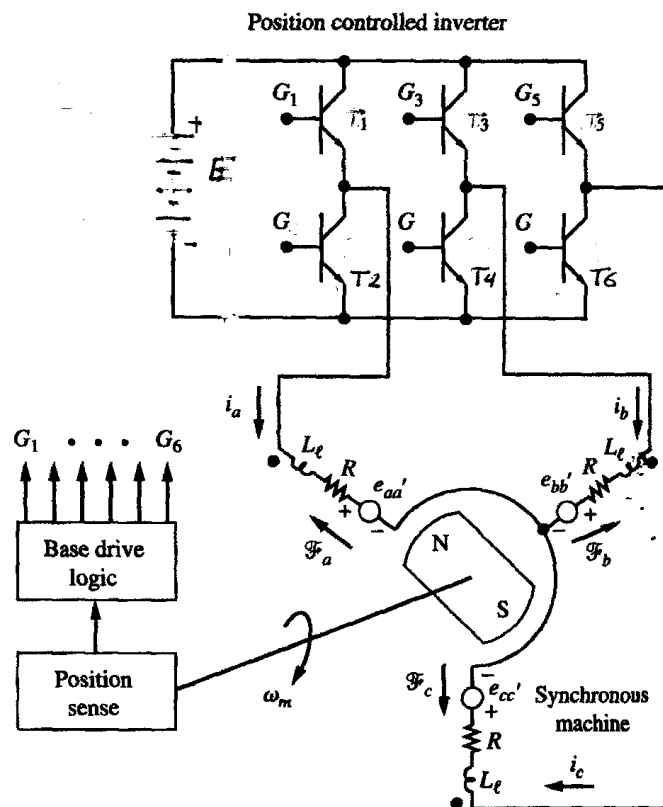


Fig.5

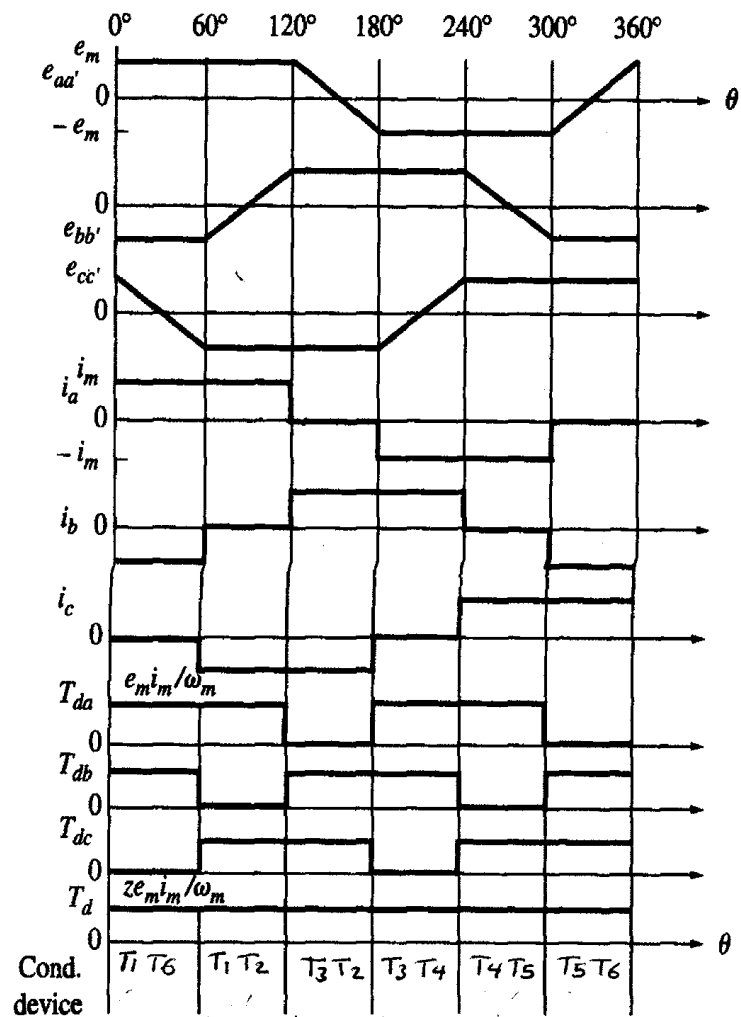


Fig.6 Timing diagram for BLDC

The control logic functions to assure the phase relationships between the coil induced trapezoidal voltages and currents shown by the timing diagram of Fig. 6, The total power converted from electrical to mechanical form is given by

$$3P_d = \sum_{a,b,c} e_k i_k = e_{aa'} i_a + e_{bb'} i_b + e_{cc'} i_c$$

The total electromagnetic developed torque at speed ω_m as follows

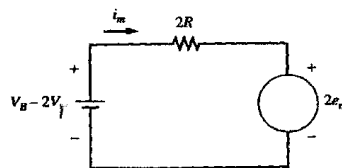
$$3T_d = \frac{3P_d}{\omega_m} = \frac{\sum_{a,b,c} e_k i_k}{\omega_m}$$

The torque contributions of each phase (T_{da} , T_{db} , T_{dc}) are easily determined with the aid of the timing diagram of Fig. 6. The individual phase contributions to developed torque are added to give total developed torque as

$$3T_d = \frac{2e_m i_m}{\omega_m}$$

As apparent from Fig. 2, flux paths of the PM synchronous machine include a radial path across the PM material which has a permeability that approaches that of free space. Consequently, the reluctance of coil coupling flux paths is large; thus, coil inductance values are small. For operating speeds that result in a frequency (f) of coil induced voltages such that $T = 1/f \gg L/R$, the coil currents are near the ideal waveforms of Fig. 6. The dc output voltage (V_B) of the rectifier of Fig. 5 at any point in time is impressed across two phase windings that present a total induced voltage of magnitude $2e_m$. The simple equivalent circuit of Fig. 7 then models the BDCM where the impressed voltage appearing at the motor terminals has been reduced by $2V_T$ the ON-state voltage drop across the two conducting transistors.

Fig.7 Equivalent circuit of BDCM



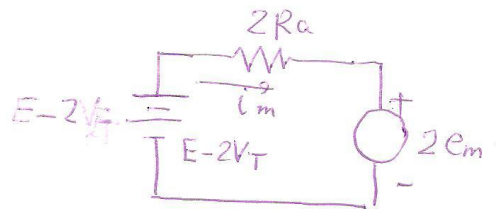
Example. 1

A small BLCD motor is operating with dc voltage $E = 325\text{V}$. The motor phase winding resistance is $2.3\ \Omega$. At a speed of 1000 rpm , the magnitude ($2e_m$) of the trapezoidal line-to-line voltage was determined to be 123V . If the BLCD is operating at 2500 rpm , find the magnitude of phase currents and the total developed torque. Assume that the forward voltage drop of a conducting transistor is 0.8V .

Solution: -

$$E = 325\text{ V}$$

$$2e_{m1} = 123\text{ V}$$



$$i_m = \frac{E - 2V_T - 2e_{m2}}{2R_a}$$

$$\frac{2e_{m1}}{2e_{m2}} = \frac{n_1}{n_2} = \frac{1000}{2500}$$

$$\therefore 2e_{m2} = 2.5 \times 123 = 307.5$$

$$\omega = \frac{2\pi n}{60} = \frac{2500}{9.55} = 261.78\text{ rad/sec.}$$

$$i_m = \frac{325 - 1.6 - 307.5}{2 \times 2.3} = \frac{325 - 309.1}{4.6} = \frac{15.9}{4.6} = 3.45\text{ A}$$

The total developed torque

$$3T_d = \frac{2e_m i_m}{\omega} = \frac{307.5 \times 3.45}{261.78} = \underline{\underline{4.06\text{ N.m}}}$$

Brushless DC Machine

Example 2: A brushless DC machine rated 12 V, 10W, has a 3-phase stator winding connected in delta. The controller of the drive is so designed to produce quasi-Square current waveform in the stator winding of 200 mA amplitude with 4 msec. duration in 6 msec. half wave. The machine has two-pole permanent magnet rotor. Sketch the controller/driver/machine configuration and calculate the frequency and the synchronous speed at which the machine will run:

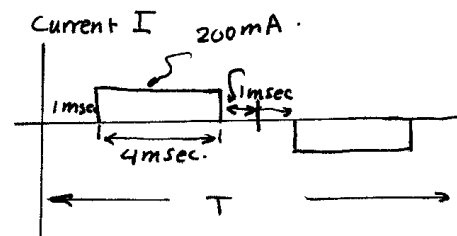
Solution:

$$T = 2 \times (1 + 4 + 1) = 12 \text{ msec.}$$

$$f = \frac{1}{T} = \frac{1}{12 \times 10^{-3}} = 83.3 \text{ Hz}$$

$$n_s = \frac{120f}{P} = \frac{120 \times 83.3}{2}$$

$$\approx 5000 \text{ r.p.m.}$$



Example 3:

For example 2, calculate the ~~ohmic motor losses~~ P_{cu} the core loss $P_{cl} = 10\text{ W}$ and the stray losses $P_s = 12\text{ W}$.

Solution:

The ohmic motor losses P_{cu} is

$$P_{cu} = 3 \dot{I}_m^2 R = 3 \times (2.64)^2 \times 2.3 = 48.24\text{ W}$$

$$\text{output power} = 3T_d \omega = 3 \times 0.97 \times 223 = 649.66\text{ W}$$

$$P_{cl} = 10\text{ W} \quad \text{and} \quad \text{stray losses } P_s = 12\text{ W}$$

$$\begin{aligned} \eta &= \frac{\text{Output}}{\text{Output} + \text{losses}} = \frac{3T_d \omega}{3T_d \omega + P_{cl} + P_s + P_{cu}} \\ &= \frac{649.66}{649.66 + 48.24 + 10 + 12} = \frac{649.66}{720.9} \\ &= 0.9011 = 90.11\% \end{aligned}$$

Small electrical Machines

Dr. M.T. Lazim

Reluctance Motor

Reluctance motor or "synchronous reluctance motor" has a salient pole configuration. The view diagram of the motor is shown in Fig.1.

When the rotor and stator are excited by separate coils produce an mmf, but only one resultant mmf can exist. If the rotor inductance L_1 can be considered constant, then the stator inductance L_2 will be variable and it will be a function of θ , see Fig.2.

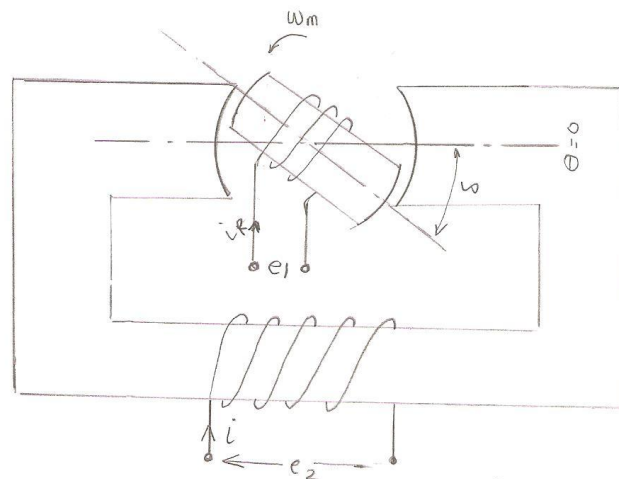


Fig.1

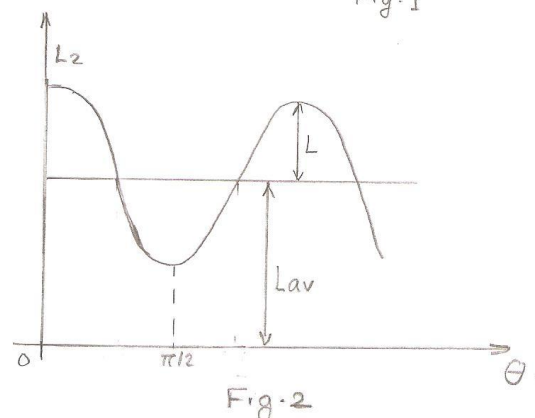


Fig.2

$$L_2 = L_{av} + L \cos 2\theta$$

At $\theta = 0$, $L_2 = \text{maximum}$
 when $\theta = 90^\circ$, $L_2 = \text{minimum}$.

The flux linkage $\lambda = L_2 i$.
 where i is the current in the stator winding

The stored energy in the field is:

$$W = \frac{1}{2} L_2 i^2$$

Let $i = I_m \sin \omega t$

The torque:

$$T_e = - \frac{\partial W}{\partial \theta} + i \frac{\partial \lambda}{\partial \theta} \quad (\text{By definition}).$$

but $\frac{\partial W}{\partial \theta} = \frac{1}{2} i^2 \frac{\partial L_2}{\partial \theta}$.

$$T_e = - \frac{1}{2} i^2 \frac{\partial L_2}{\partial \theta} + i^2 \frac{\partial L_2}{\partial \theta} = \frac{1}{2} i^2 \frac{\partial L_2}{\partial \theta}.$$

$$= \frac{1}{2} (I_m \sin \omega t)^2 [2 L (-\sin 2\theta)]$$

$$= - I_m^2 L \sin^2 \omega t \sin 2\theta.$$

$$\sin^2 \omega t = \frac{1}{2} (1 - \cos 2\omega t)$$

$$= - \frac{1}{2} I_m^2 L [(1 - \cos 2\omega t) \sin 2\theta]$$

Let $\theta = \omega_m t - \delta$ where δ is any starting angle from the reference angle of θ_0 (as shown in Fig-1).
 ω_m is the rotational speed.

$$\begin{aligned} T_e &= - \frac{1}{2} I_m^2 L [\sin 2(\omega_m t - \delta) - \cos 2\omega t \sin 2(\omega_m t - \delta)] \\ &= - \frac{1}{2} I_m^2 L [\sin 2(\omega_m t - \delta) - \frac{1}{2} \{ \sin 2(\omega_m t - \omega t - \delta) \\ &\quad + \sin 2(\omega_m t + \omega t - \delta) \}]. \end{aligned}$$

Assuming that $\omega = \omega_m$, i.e., the rotor turns at synchronous speed:

$$\begin{aligned} T_e &= - \frac{1}{2} I_m^2 L [\sin 2(\omega_m t - \delta) - \frac{1}{2} \{ \sin 2(\omega_m t - \omega_m t - \delta) \\ &\quad + \sin 2(\omega_m t + \omega_m t - \delta) \}] \\ &= - \frac{1}{2} I_m^2 L \left\{ \frac{1}{2} \sin 2\delta + \sin 2(\omega_m t - \delta) + \sin 2(2\omega_m t - \delta) \right\}. \end{aligned}$$

The time average of this expression is

$$T_{ave} = - \frac{1}{4} I_m^2 L \sin 2\delta.$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

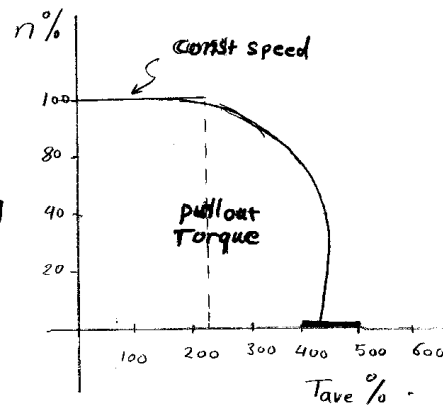
The torque-speed characteristics of synchronous reluctance motor.

The reluctance motor is not self-starting, but it has self-sustaining at synchronous speed. Continuous rotation results from alternating electromagnetic and inertia torques.

The speed remains constant even for torques which is double the rated torque of the motor. Hence the constant speed characteristic of reluctance

motors, and their simple construction, ruggedness and particularly low cost make them desirable for:

- Timing devices.
- Turntables
- Tape recorders
- Food processing and packing industries.
- Synthetic textile industry.



Speed-torque characteristics

Critical torque angle δ :

If the mechanical load on the motor shaft is increased to a value that causes $\delta > 45^\circ$, the motor will lose synchronism. This value of δ is called the critical torque angle. Hence the critical torque (pull out torque) is

$$\begin{aligned} T_{av, \text{critical}} &= -\frac{1}{4} I_m^2 L \sin 2\delta_{\text{critical}} \\ &= -\frac{1}{4} I_m^2 L \sin 90^\circ = -\frac{I_m^2}{4} L \end{aligned}$$

Example 1 :

A 2-pole small reluctance motor has the following parameters :

Stator inductance $L_2 = 0.5 + 0.35 \cos 2\theta$

Rotor inductance $L_1 = 0.25 \text{ H.}$

Speed $\omega = 314 \text{ rad/sec.}$

Assume an AC input current to the stator of $i_s = 20 \sin 314t$, and a d.c. current to the rotor $i_R = 3 \text{ A}$, calculate the average torque produced at starting angle $\delta = 30^\circ$.

Solution:

$$\omega_m = 314 \text{ rad/sec.}, I_m = 20 \text{ A}$$

$$L = 0.35 \text{ H}$$

$$T_e = -\frac{1}{2}(20)^2(0.35) \left[\frac{1}{2} \sin 2\left(\frac{\pi}{6}\right) + \sin(314t - \frac{\pi}{6}) + \sin(4 \times 314t - \frac{\pi}{3}) \right]$$

$$= -70 \left[\frac{1}{2} \left(\frac{\sqrt{3}}{2} \right) + \sin(314t - \frac{\pi}{6}) + \sin(1256t - \frac{\pi}{3}) \right]$$

$$= -30.31 + \sin(314t - \frac{\pi}{6}) + \sin(1256t - \frac{\pi}{3})$$

$$\therefore T_{av} = \underline{\underline{-30.31 \text{ N.m.}}}$$

Example 2: A 1.5 kW, 4-pole, 240 V, 50 Hz, small reluctance motor, operating under rated load conditions, has a torque angle of 30° . Determine:

- Torque load on the shaft (average torque)
- Torque angle if the supply voltage drop to 224 V.
- Will the rotor pull out of synchronism?

Solution

$$(a) \quad n_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm.}$$

$$P_{\text{shaft}} = T_{\text{av}} \cdot \omega_s = T_{\text{av}} \cdot \frac{2\pi n_s}{60} = 1500$$

$$\therefore T_{\text{av}} = \frac{1500 \times 60}{2\pi \times 1500} = \frac{30}{\pi} = 9.55 \text{ N.m.}$$

$$(b) \quad I_m = \sqrt{2} I = \sqrt{2} \frac{V}{Z}$$

$$T_{\text{av}} = -\frac{1}{4} I_m^2 L \sin 2\delta$$

$$= -\frac{1}{4} \left(\sqrt{2} \frac{V}{Z} \right)^2 L \sin 2\delta$$

If same torque and frequency

$$T_{\text{av}1} = T_{\text{av}2}$$

$$-\frac{1}{4} \left(\sqrt{2} \frac{V_1}{Z} \right)^2 L \sin 2\delta_1 = -\frac{1}{4} \left(\sqrt{2} \frac{V_2}{Z} \right)^2 L \sin 2\delta_2$$

$$V_1^2 \sin 2\delta_1 = V_2^2 \sin 2\delta_2$$

$$(240)^2 \sin(2 \times 30) = (224)^2 \sin 2\delta_2$$

$$\therefore \sin 2\delta_2 = \left(\frac{240}{224} \right)^2 \sin 60$$

$$= 0.994$$

$$2\delta_2 = \sin^{-1} 0.994 = 83.72$$

$$\therefore \delta_2 = 41.86^\circ$$

- Because $\delta_2 < 45^\circ$, the rotor will not lose (pull-out of) synchronism.

STEPPER MOTORS

A stepper motor rotates by a specific number of degrees in response to an input electrical pulse. Typical step sizes are 2° , 2.5° , 5° , 7.5° , and 15° for each electrical pulse. The stepper motor is an electromagnetic incremental actuator that can convert digital pulse inputs to analog output shaft motion. It is therefore used in digital control systems. A train of pulses is made to turn the shaft of the motor by steps. Typical applications of stepper motors requiring incremental motion are printers, tape drives, disk drives, machine tools, process control systems, X—Y recorders, and robotics.

Two types of stepper motors are widely used:

- (1) the variable-reluctance.
- (2) the permanent magnet.

1.VARIABLE-RELUCTANCE STEPPER MOTOR

A variable-reluctance stepper motor can be of the single-stack type or the multiple-stack type .

Single-Stack Stepper Motor

A basic circuit configuration of a four-phase, two-pole, single-stack, variable- reluctance stepper motor is shown in Fig.1. When the stator phases are excited with dc current in proper sequence, the resultant air gap field steps around and the rotor follows the axis of the air gap field by virtue of reluctance torque. This reluctance torque is generated because of the tendency of the ferromagnetic rotor to align itself along the direction of the resultant magnetic field .

Figure 2 shows the mode of operation for a 45° step in the clockwise direction. The windings are energized in the sequence A, A + B, B, B + C

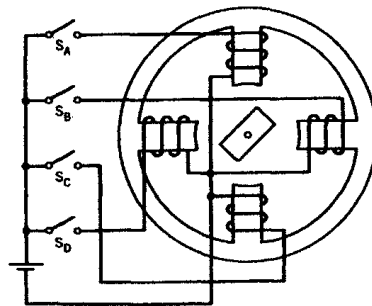


Fig.1

and so forth, and this sequence is repeated. When winding A is excited, the rotor aligns with the axis of phase A. Next, both windings A and B are excited, which makes the resultant mmf axis move 45° in the clockwise direction. The rotor aligns with this resultant mmf axis. Thus, at each transition the rotor moves through 45° as the resultant field is switched around. The direction of rotation can be reversed by reversing the sequence of switching the windings, that is, A, A + D, D, D + C, etc

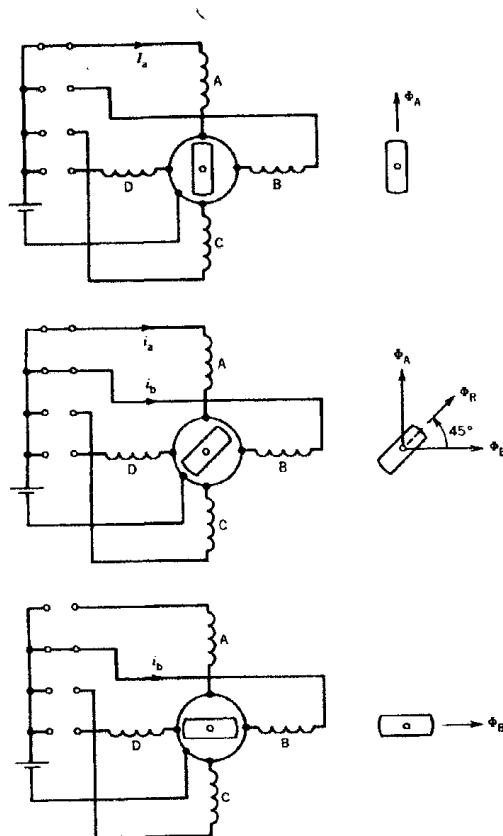
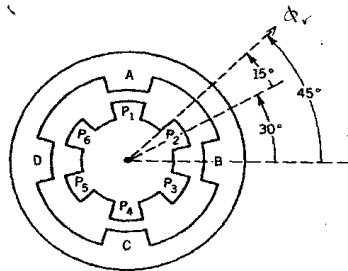


Fig.2

A multipole rotor construction is required in order to obtain smaller step sizes. The construction of a four-phase, six-pole stepper motor is shown

in Fig.3. When phase A winding is excited, pole P₁ is aligned with the axis of phase A, as shown in Fig.3. Next, phase A and phase B windings are excited. The resultant mmf axis moves in the clockwise direction by 45° and pole P₂, nearest to this new resultant field axis, is pulled to align with it. The motor therefore steps in the anticlockwise direction by 15°. Next, phase A winding is de-excited and the excitation of phase B winding pulls pole P₃ to align with the axis of phase B. Therefore, if the windings are excited in the sequence A, A + B, B, B + C, C the rotor rotates in steps of 15° in the anticlockwise direction .

Fig.3



Multistack Stepper Motor

Multistack variable-reluctance-type stepper motors are widely used to give smaller step sizes. The motor is divided along its axial length into magnetically isolated sections ("stacks"), and each of these sections can be excited by a separate winding ("phase"). Three-phase arrangements are most common, but motors with up to seven stacks and phases are available.

Figure 4 shows the longitudinal cross section (i.e., parallel to the shaft) of a 3-stack (3-phase) variable-reluctance stepper motor. The stator of each stack has a number of poles. Figure 5 shows an example with four poles. Adjacent poles are wound in the opposite sense, and this produces four main flux paths, as shown in Fig. 5. Both stator and rotor have the same number of teeth (12 in Fig. 5a). Therefore, when a particular phase is excited, the position of the rotor relative to the stator in that stack is accurately defined, as shown in Fig. 5a. The rotor teeth in each stack are aligned, whereas the stator teeth have a different orientation between stacks, as shown in the developed diagram of rotor and stator teeth in Fig. 5b. Therefore, when stack A is energized, the rotor and stator teeth in stack A are aligned but those in stacks B and C are not aligned, as shown in Fig. 5b. Next, if excitation is changed from stack A to stack B, the

stator and rotor teeth in stack B are aligned.

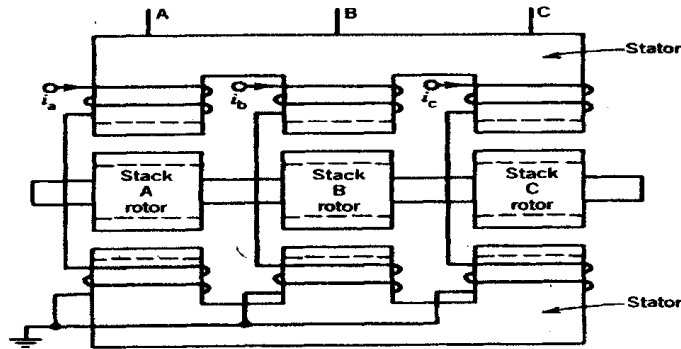


Fig.4

Another step motion in the clockwise direction can be obtained if excitation is changed from stack B to stack C. Another change of excitation from stack C to stack A will once more align the stator and rotor teeth in stack A. However, during this process (A — B — C — A) the rotor has moved one rotor tooth pitch, that is, the angle between adjacent rotor teeth. $\left(\frac{360}{x} = 10^\circ\right) \quad \frac{360}{12 \times 3} = 10$

Let x be the number of rotor teeth and N the number of stacks or phases. Then

$$\text{Tooth pitch} \quad \tau_p = 360 / x$$

$$\text{Step size} \quad \Delta\theta = 360 / xN$$

Number of step per revolution :

$$n = 360 / \Delta\theta = xN$$

For the motor illustrated in Fig.5

$$\tau_p = 360 / 12 = 30^\circ$$

$$\Delta\theta = 360 / 12 \times 3 = 10^\circ$$

$$n = 360 / 10 = 36$$

Typical step sizes for the multistack variable-reluctance stepping motor are in the range 2 to 15 degrees.

Static Torque Characteristics

The torque developed per step is given by:

$$T = T_{PK} \sin \alpha \theta$$

$$= \frac{1}{2} (i^2 L_{\theta} \sin \alpha \theta) \times N$$

where T_{PK} = peak static torque .

i = current per phase .

L_{θ} = inductance per phase = $L_{ave} + L \cos \alpha \theta$

N = No. of phases (stacks)

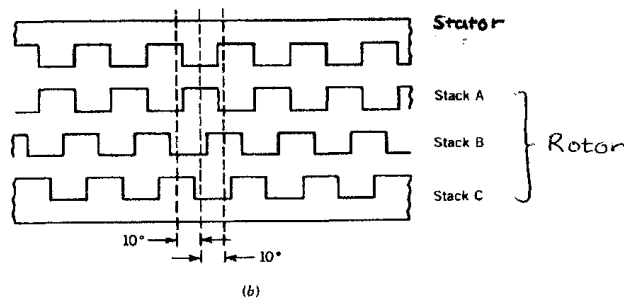
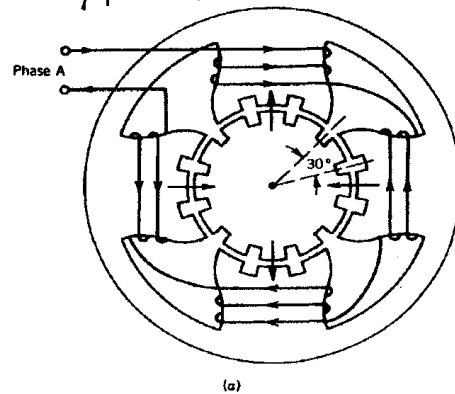


FIGURE 5

Teeth position in a four-pole, three- stack, variable-reluctance stepper motor. (a) Phase A excited. Rotor and stator teeth are aligned. (b) Developed diagram for rotor and stator teeth for phase A excitation.

Stepper Motor

Example 1.

A three-stack variable reluctance stepper motor has parameters as follows:

$$L_{\theta} = 15 + 5 \cos x \theta \text{ mH} \dots \text{inductance per phase.}$$

$$x = 10 \dots \text{number of rotor and stator teeth}$$

$$i = 2\text{A} \dots \text{current per phase.}$$

It is required to calculate:

- (a) step length (step size) and the tooth pitch.
- (b) inductance of the individual phases, torque, and to draw the torque-angle characteristic.
- (c) Average value of revolution per minute when 100 pulses per seconds are brought up to the input.

Solution:

- (a) The number of phases = number of stacks
 $\therefore N = 3$.

$$\text{Tooth pitch } \tau_p = \frac{360^\circ}{\text{No. of teeth}} = \frac{360^\circ}{x} = \frac{360^\circ}{10} = 36^\circ.$$

$$\text{Step size } \Delta\theta = \frac{360^\circ}{xN} = \frac{360^\circ}{10 \times 3} = 12^\circ.$$

(b) $L_{\theta} = 15 + 5 \cos x \theta = 15 + 5 \cos 10 \theta$.

For all phases, the torque is given by:

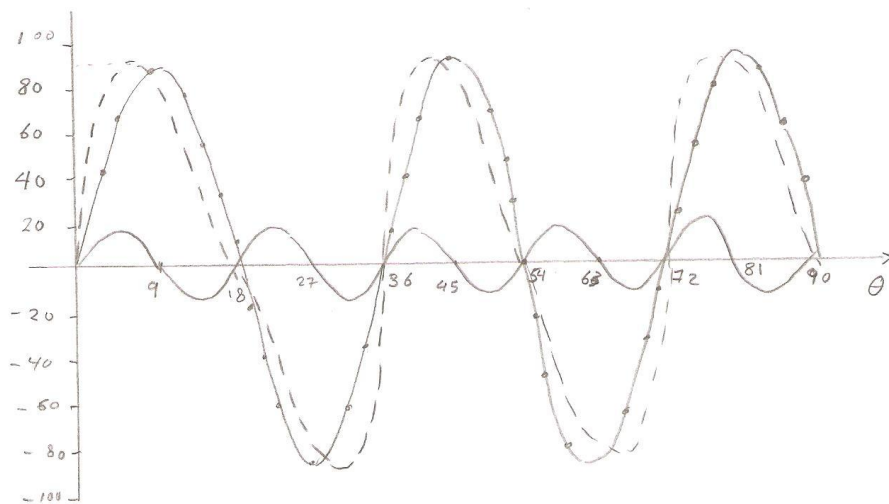
$$\begin{aligned} T &= \frac{1}{2} i^2 L_{\theta} N \sin x \theta \\ &= \frac{1}{2} (2)^2 (15 + 5 \cos 10 \theta) \times 3 \sin 10 \theta \\ &= 6 (15 + 5 \cos 10 \theta) \sin 10 \theta. \end{aligned}$$

To plot this torque we can simplify the above equation as :

$$T = 90 \sin 10\theta + 30 \cos 10\theta \sin 10\theta$$

$$= 90 \sin 10\theta + 15 \sin 20\theta. \quad (\sin 2\theta = 2 \sin\theta \cos\theta).$$

The addition is performed graphically :



Torque- θ ch/s.

—•—•—•— 90 sin 10 θ
 ————— 15 sin 20 θ
 - - - - - Resultant torque.

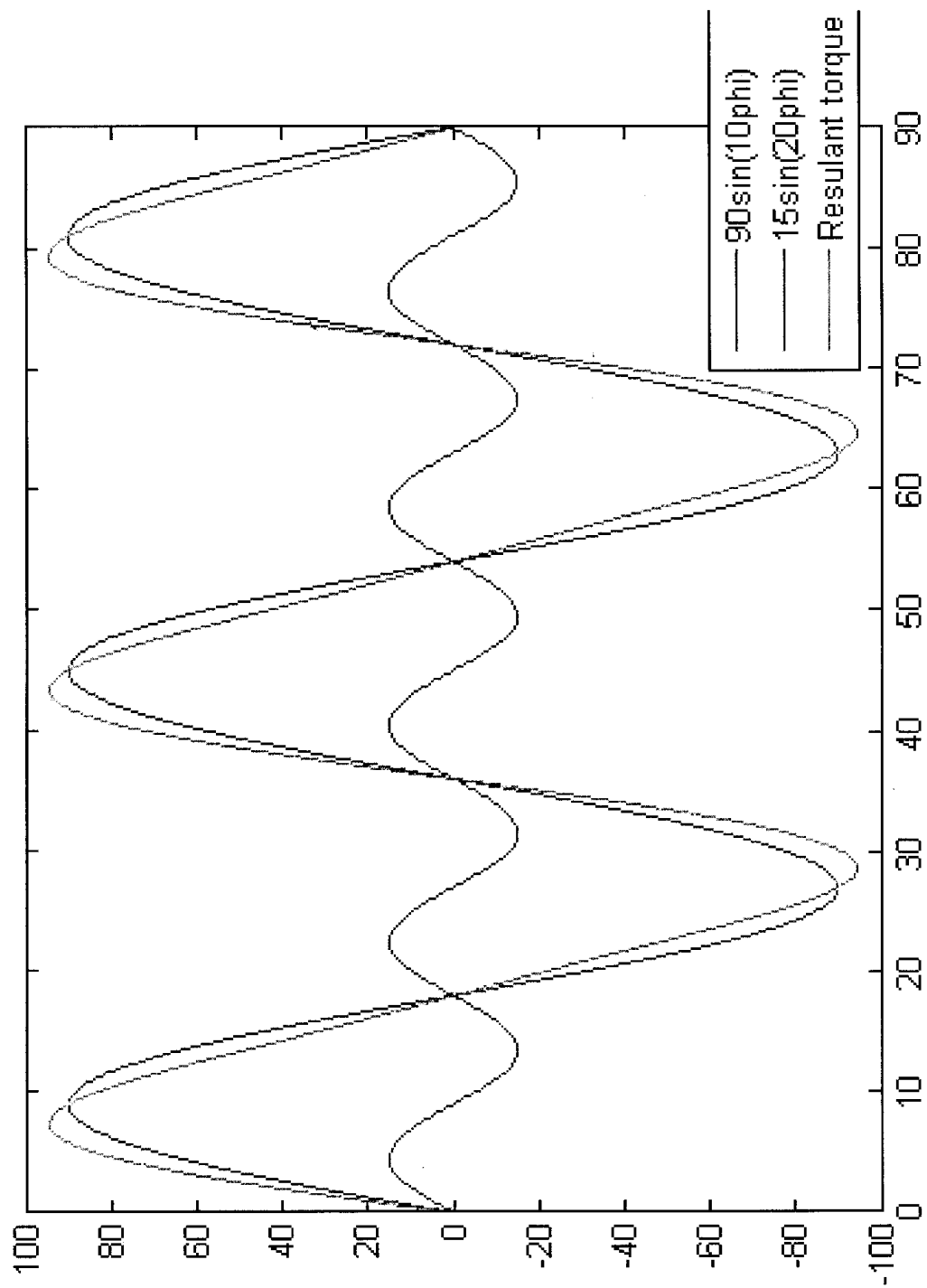
(c) If the ifp is 100 pulse/sec:

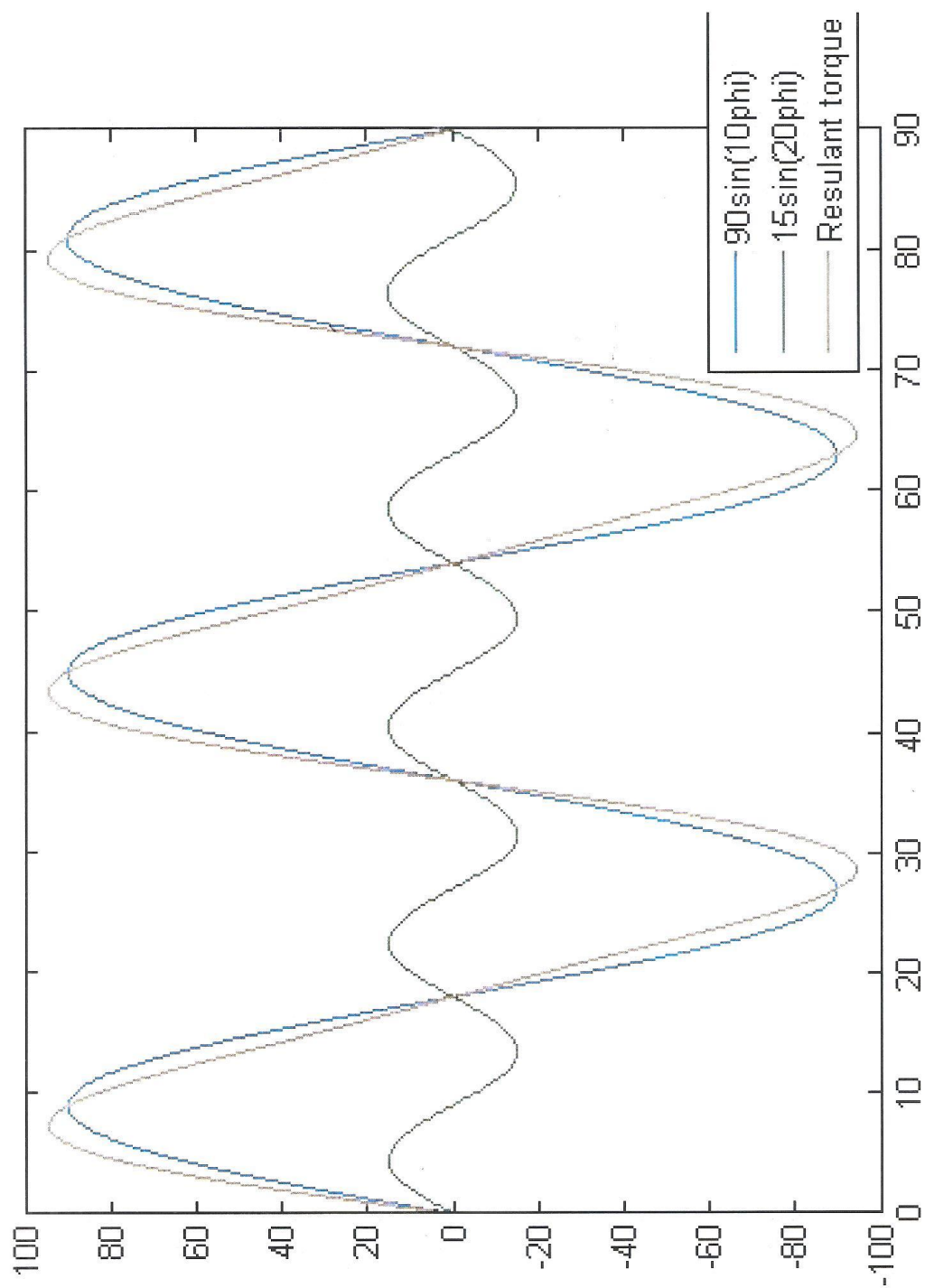
Speed = 100 pulse/sec $\times$ step size (12 degree)
 (because each pulse represents one step).

$$\text{Speed} = 100 \times 12 = 1200 \text{ }^\circ/\text{sec} = \frac{1200}{360} \text{ rev/sec}$$

now to change into rpm we have: $= \frac{1200 \times 60}{360} \text{ rev/min.}$

$$\text{Speed}_{\text{rpm}} = \frac{60}{360} \text{ Speed}_{^\circ/\text{sec}} = \frac{60}{360} \times 1200 = 200 \text{ r.p.m.}$$





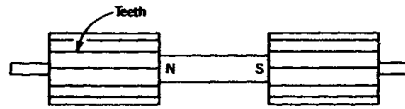


Fig. 7 Rotor of hybrid stepper motor .

DRIVE CIRCUITS

The command signals for a stepper motor are normally obtained from low- power logic circuits that are built with TTL or CMOS digital integrated circuits (ICs). The driving current available is either 20 mA at 5 V (TTL) or 1 mA at 5—15 V (CMOS). However, a typical variable-reluctance stepper motor producing a torque of 1.2 N . m has a rated winding excitation of 5 V and 3A. Therefore power amplification stages are required between the low-power command signals and the high-power stepper motors.

Variable-reluctance stepper motors require more than two phases (three phases are typical). The phase currents need only be switched on or off and current polarity is irrelevant for torque production. Permanent magnet stepper motors require two phases, and the current polarity is important.

Unipolar Drive Circuit

Figure 8 shows a simple unipolar drive circuit suitable for a three-phase variable-reluctance stepper motor. Each phase winding is excited by a separate drive circuit. The main switching device is a transistor. A phase winding is excited by applying a control signal to the base of the transistor.

In order to excite a phase winding, a sufficiently high base current is passed through the base of the transistor. The supply voltage V appears across the phase winding and the resistance R_{ext} connected in series with the winding. The dc supply voltage V is chosen so that it produces the rated current I in the winding.

$$V_s = I(R_w + R_{ext})$$

$$I = \frac{V_s}{R_w + R_{ext}}$$

where R_w is the phase winding resistance. The phase winding has a large inductance and therefore the electrical time constant is large. As a result, buildup of current in the phase winding to its rated value is slow, causing

unsatisfactory operation of the motor at high stepping rates. The addition of the external resistance R_{ext} decreases the electrical time constant, thereby speeding up the current buildup.

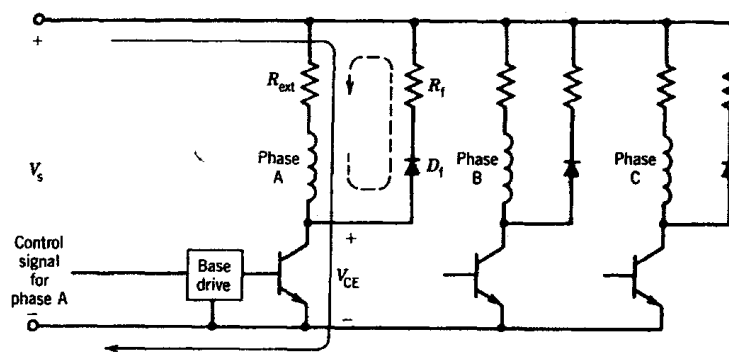


FIGURE 8 Unipolar drive circuit for a three-phase variable-reluctance stepper motor

When the base drive current is removed to switch off the transistor, a large induced voltage will appear across the transistor if the winding current is suddenly interrupted. The large voltage may permanently damage the transistor. This possibility is avoided by providing an alternative path for the phase winding current—known as a freewheeling path. Therefore, when the transistor is switched off, the phase winding current will continue to flow in the freewheeling diode D_1 and a freewheeling resistance R_1 . The maximum voltage across the transistor occurs at the instant of switch off and is

$$V_{CE(max)} = V_s + I R_1$$

Subsequently, the phase current will decay in the closed circuit formed by the phase winding, D_1 , R_1 , and R_{ext} . The magnetic energy stored in the phase inductance at turnoff of the transistor is dissipated in the resistances of this closed circuit.

PERMANENT MAGNET STEPPER MOTOR

The permanent magnet stepper motor has a stator construction similar to that of the single-stack variable-reluctance type, but the rotor is made of a permanent magnet material. Figure 6 shows a two-pole, permanent magnet stepper motor. The rotor poles align with two stator teeth (or poles) according to the winding excitation. Figure 6 shows the alignment if phase A winding is excited. If the excitation is switched to phase B, the rotor moves by a step of 90° . Note that current polarity is important in the permanent magnet stepper motor, because it decides the direction in which the motor will move. Figure 6 illustrates the rotor position for positive current in phase A. A switchover to positive current in phase B winding will produce a clockwise step, whereas a negative current in phase B winding will produce an anticlockwise step. It is difficult to make a small permanent magnet rotor with a large number of poles, and therefore stepper motors of this type are restricted to larger step sizes in the range 30 to 90 degrees.

Permanent magnet stepper motors have higher inertia and therefore slower acceleration than variable-reluctance stepper motors. The maximum step rate for permanent magnet stepper motors is 300 pulses per second, whereas it can be as high as 1200 pulses per second for variable reluctance stepper motors. The permanent magnet stepper motor produces more torque per ampere stator current than the variable-reluctance stepper motor .

Hybrid stepper motors are also commercially available in which the rotor has an axial permanent magnet at the middle and ferromagnetic teeth at the outer sections as shown in Fig7. Smaller step sizes can be obtained from these motors, but they are more expensive than the variable-reluctance- type stepper motors.

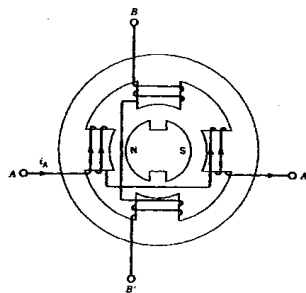
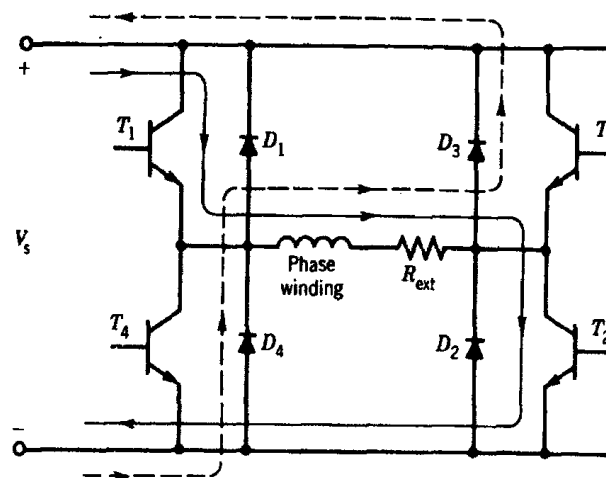


Fig. 6 Permanent magnet stepper motor .

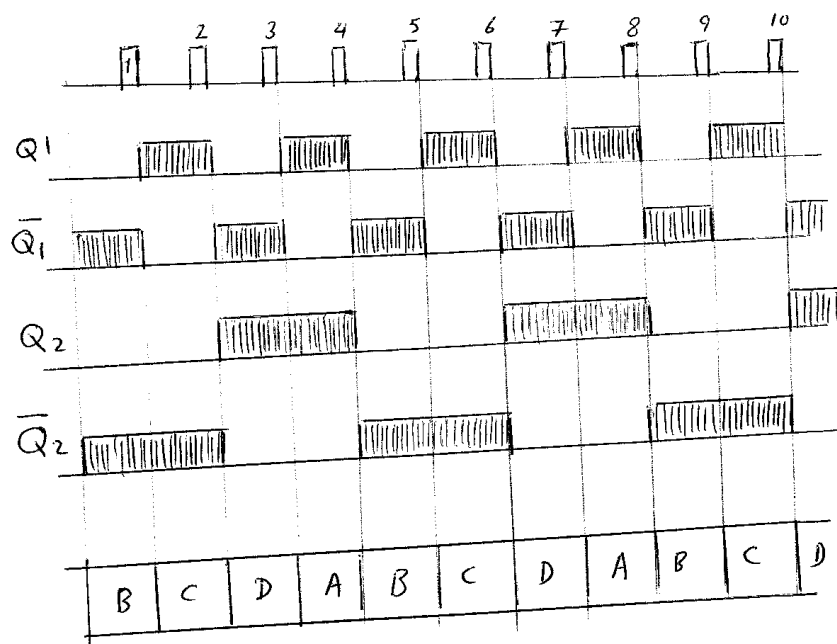
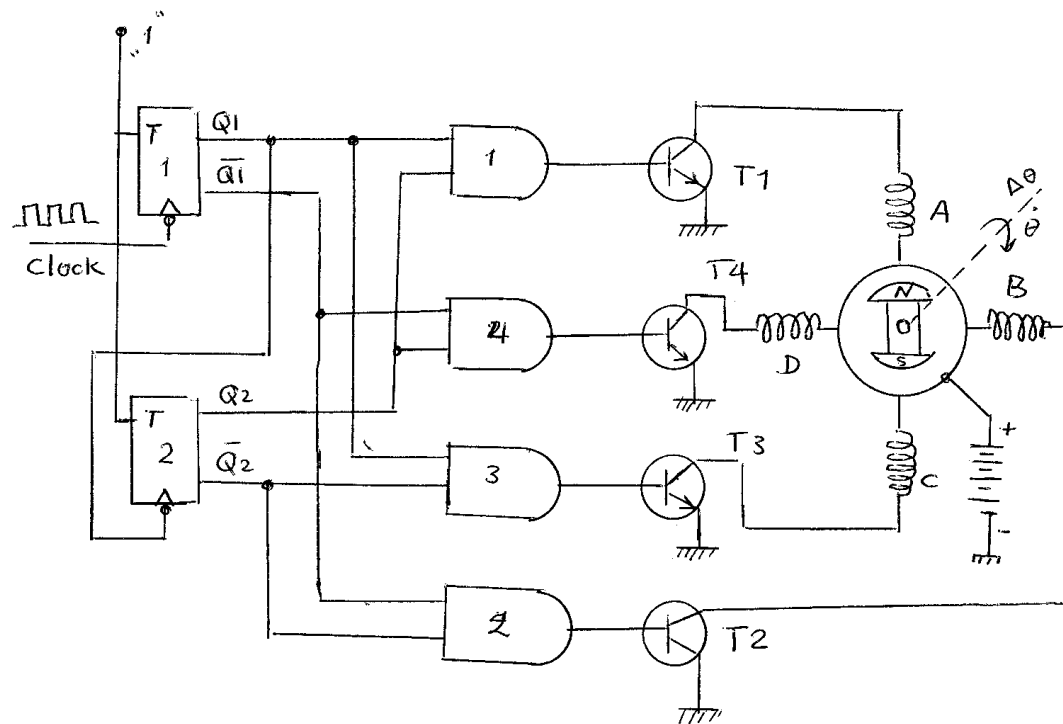
Bipolar Drive Circuit

Figure 9 shows one phase of a bipolar drive circuit suitable for a permanent magnet or hybrid-type stepper motor. The transistors are switched in pairs according to the current polarity required for the phase winding. For example, transistors T1 and T2 are turned on simultaneously so that current can flow from left to right in the phase winding, as shown in Fig.9. If transistors T3 and T4 are turned on simultaneously, current will flow in the opposite direction. The four diodes D1 to D4 connected in antiparallel with the switching transistors provide the paths for the freewheeling currents.

For example when T1 and T2 are switched on, current flows from dc supply to T1, phase winding (left to right), T2, and back to dc supply. When T1 and T2 are switched off (by removing their base currents) current in the phase winding cannot decay instantaneously because of winding inductance. The current therefore flows through diodes D3 and D4 to the dc supply, as shown in Fig. 9 by dashed lines. Note that when current flows through D3 and D4 to the dc supply, some of the energy stored in the phase winding inductance at turnoff (of the transistors) is returned to the supply.



.FIGURE 9 One phase of a bipolar drive circuit



Waveforms for $Q_1, \bar{Q}_1, Q_2, \bar{Q}_2, A, B, C, D$ phases for the first 10 clock pulses with the aim to determine the sequence of phase excitations

Stepper Motor Drive Circuit

Example 1: A 3-phase variable-reluctance stepper motor has the following parameters:

$R_w = 1 \Omega$, $L_w = 30 \text{ mH}$... (average phase winding inductance).
 $I = 3 \text{ A}$.. rated winding current.

Design a simple unipolar drive circuit such that the electrical time constant is 2 msec at phase turn-on and 1 msec at turnoff. The stepping rate is 300 step per second.

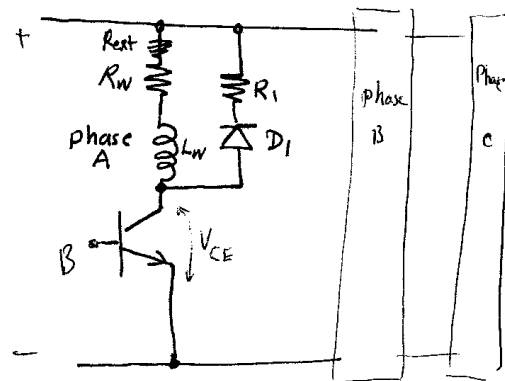
Solution

Turnon time constant

$$\tau_{on} = \frac{L_w}{R_w + R_{ext}}$$

$$\therefore R_w + R_{ext} = \frac{L_w}{\tau_{on}} = \frac{30 \times 10^{-3}}{2 \times 10^{-3}} = 15 \Omega.$$

$$- R_{ext} = 15 - R_w = 15 - 1 = 14 \Omega.$$



- This resistance must be able to dissipate the power lost when rated current flows through the phase winding continuously, i.e.:

$$P_{R_{ext}} = I^2 R_{ext} = (3)^2 \times 14 = 126 \text{ W}.$$

- The required d.c. supply,

$$V_s = I(R_w + R_{ext}) = 3 \times (15) = 45 \text{ Volts}.$$

- The turnoff time constant

$$\tau_{off} = \frac{L_w}{R_w + R_{ext} + R_f}$$

$$R_w + R_{ext} + R_1 = \frac{30 \times 10^{-3}}{1 \times 10^3} = 30 \Omega.$$

$$\therefore R_1 = 30 - 15 = 15 \Omega.$$

Energy stored in the inductance L_w of the phase winding at turnoff $= \frac{1}{2} L_w I^2 = \frac{1}{2} \times 30 \times 10^{-3} \times (3)^2$
 $= 0.135 \text{ Joules}.$

The energy dissipated in $R_1 = \frac{15}{30} \times 0.135 \text{ Joules}$
 $= 0.0675 \text{ Joules}.$

- Stepping rate = 300 Step/sec.

Number of turn off in each phases $= \frac{300}{3} = 100$ per second.

Average power dissipated in $R_1 = 100 \times 0.0675 \frac{\text{J}}{\text{s}}$
 $= 6.75 \text{ W}.$

When the transistor conducts, the reverse voltage across the diode D_1 is $V_s = 45 \text{ V}$. The peak current of the freewheeling diode is 3A, which is the phase winding current at the instant the transistor turns off.

$$V_{CE(max)} = V_s + IR_1$$

$$= 45 + 3 \times 15 = 90 \text{ V}.$$

- Current rating of the transistor is 3A
 type of transistor selected:

Also	Power transistor type BJT	$\frac{3 \text{ A}}{3 \text{ A}}$	$\frac{100 \text{ V}}{100 \text{ V}}$
	Power diode rating :	3 A	100 V

Small Electrical Machines

Dr. M. T. Lazim

Tutorial sheet No. 5

Q 1. Derive an expression for the average torque of 2-pole reluctance motor in terms of direct axis inductance and rotor position δ at $t = 0$.

Q2 a) Explain the operation of a stepper motor .

b) For a 4-phase stepper with. Permanent magnet rotor, a 4 bit digital signal is used for control from a microprocessor. The 4 bits represent 4 phases starting from a to d respectively in sequence . The binary number 1 represents that the winding is excited, and a 0 represents that the winding is not excited. An output 0011 represents that the windings c and d are excited.

i) make a table of 4 bit signals which corresponds to rotor angular position 0 , 45 , 90 , ...360 degrees.

ii) For the above control sequence, what is the time interval required- between any rotor positions in sequence if the speed of the rotor is maintained at 2000 rpm?

Q3. The motor in Fig. 1 is controlled by a 4-bit digital signal from a microcomputer.

The 4 bits represent excitation of phases A to D, respectively. For example, a digital signal 1000 will cause excitation of phase A and 0110 will cause excitation of phase B and phase C.

(a) Write a table for the 4-bit digital signals for 45° step rotation. Show the angle of rotation and phases excited

(b) Continuous sequencing of the digital signals of part (a) causes the motor rotate at constant speed. Determine the number of signals per second i.e., (nibbles/sec) if the motor rotates at 720 rpm.

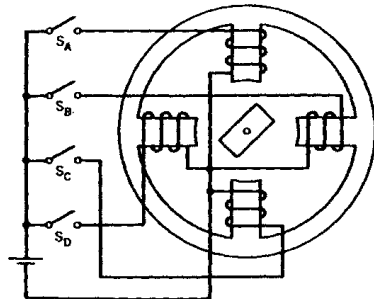


Fig.1

Q 4. A four-phase, permanent magnet, stepper motor is shown in Fig.2. The phases are excited by a 4-bit digital signal from a microcomputer

(a) Write a table for the 4-bit digital signals for a 90° step revolution. Show the angle of rotation and the phases excited.

- (b) Determine the rpm of the stepper motor if the number of pulses per second is 100.

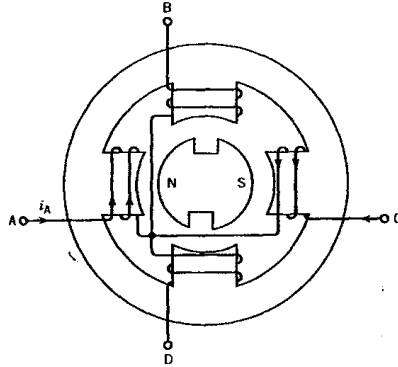


Fig .2

Q5. A three-stack, four-pole stepper motor has eight teeth on the rotor as well as on the stator. Determine the step size as excitation is changed from one stack to the next.

Q6. A single-stack, four-phase (stator) multipole stepper motor is required to produce an 18° step motion. Determine the number of rotor poles and the sequence of excitation of the stator phases. Draw a cross-sectional view of the stepper motor.

Q7. A single-stack, eight-phase (stator), multipole, stepper motor has six rotor teeth. The phases are excited one at a time

- Determine the step size
- Determine the steps per revolution.
- Determine the rpm, if the excitation frequency is 120 Hz.

Q 8. A 3- phase variable-reluctance stepper motor has the following parameters

$$R_w = 2.0 \text{ ohms}$$

$$L_w = 50 \text{ mH (average)}$$

$$I = 5 \text{ A (rated winding current)}$$

Each phase is controlled by a unipolar drive circuit. The resistance connected in series with the winding is $R_{ext} = 10 \text{ ohms}$ and that with the freewheeling diode is $R_f = 5 \text{ ohms}$.

- Determine the electrical time constants (τ_{on} and τ_{off}) at turn-on and turnoff of a phase
- Determine the value of the supply voltage V
- Determine the voltage and current ratings of the transistor and diode
- If phase current conducts for $3(\tau_{on} + \tau_{off})$, determine the maximum value of the stepping rate (steps per second) of the motor.

Small & Micro Machines	Dr. Mohammed Tawfeeq
Sheet No. 5	Problem Solution Page: 1

Problem. 2.

$$n = 2000 \text{ rpm}$$

$$\omega = \frac{2\pi n}{60} = \frac{n}{9.55}$$

$$= \frac{2000}{9.55} = 209.49 \text{ rad/sec}$$

$$\Delta\theta = \frac{\pi}{4} = 0.785 \text{ rad.}$$

$$\therefore t = \frac{\Delta\theta}{\omega} = \frac{0.785 \text{ rad}}{209.4 \text{ rad/sec}}$$

$$= 0.0374 \text{ sec} = 3.74 \text{ msec.}$$

θ	A	B	C	D
0°	1	0	0	0
45°	1	1	0	0
90°	0	1	0	0
135°	0	1	1	0
180°	0	0	1	0
225°	0	0	1	1
270°	0	0	0	1
315°	1	0	0	1
360°	1	0	0	0

Problem 3

$$(b) \quad \Delta\theta = 45^\circ$$

$$\text{Speed} \Big|_{\text{degree/sec}} = (\text{Pulse/sec}) \Delta\theta$$

$$\text{Speed} \Big|_{\text{rpm}} = \frac{60}{360} \text{Speed} \Big|_{\text{degree/sec}}$$

$$750 = \frac{60}{360} \times \text{Speed} \Big|_{\text{degree/sec}}$$

$$\therefore \text{Speed} \Big|_{\text{deg./sec}} = \frac{360 \times 750}{60} = 4500$$

$$\therefore \text{Speed} \Big|_{\text{deg/sec}} = \text{puls/sec (nibbles/sec)} \cdot \Delta\theta$$

$$\text{Hence } 4500 = y \Delta\theta \quad \text{or } y = \frac{4500}{45} = 100 \text{ nibble/sec.}$$

angle	(a) phases				Signal
θ	A	B	C	D	
0°	1	0	0	0	1 0 0 0
45°	1	1	0	0	1 1 0 0
90°	0	1	0	0	0 1 0 0
135°	0	1	1	0	0 1 1 0
180°	0	0	1	0	0 0 1 0
225°	0	0	1	1	0 0 1 1
270°	0	0	0	1	0 0 0 1
315°	1	0	0	1	1 0 0 1
360°	1	0	0	0	1 0 0 0

- 2 -

problem 4

$$\Delta\theta = 45^\circ$$

$$\text{Speed}_{\text{deg/sec}} = (\text{pulse/sec}) \Delta\theta$$

$$= 100 \times 45$$

$$= 4500$$

$$\text{Speed}_{\text{rpm}} = \frac{60}{360} \text{ Speed}_{\text{deg/sec}}$$

$$= \frac{60}{360} \times 4500$$

$$= 750 \text{ rpm}$$

problem 5

$$N = \text{no. of stacks} = \text{no. of phases} = 3$$

$$x = \text{no. of teeth of the rotor} = 8$$

$$\Delta\theta = \frac{360}{xN} = \frac{360}{3 \times 8} = 15^\circ \quad \underline{\text{Ans}}$$

Problem 6

$$\Delta\theta = 18^\circ$$

For single-stack motor

$$\Delta\theta = \frac{360}{x}$$

$$\therefore x = \frac{360}{\Delta\theta} = \frac{360}{18} = 20 \quad \text{no. of poles on rotor.}$$

problem 7

- 3 -

(a) $N = 8$ = no. of stator phases

$$x = 6$$

$$\Delta\theta = \frac{360}{Nx} = \frac{360}{8 \times 6} = 7.5^\circ$$

(b) Number of steps per revolution is

$$n = \frac{360}{\Delta\theta} = xN = 48$$

(c) $f = 120$ Hz = 120 cycle/sec = 120 pulse/sec.

$$\text{Speed} \Big|_{\text{degree/sec}} = 120 \times (\Delta\theta) = 120 \times 7.5^\circ = 900$$

$$\text{Speed} \Big|_{\text{rev/min}} = \frac{60}{360} \times 900 = 150 \text{ rpm}$$

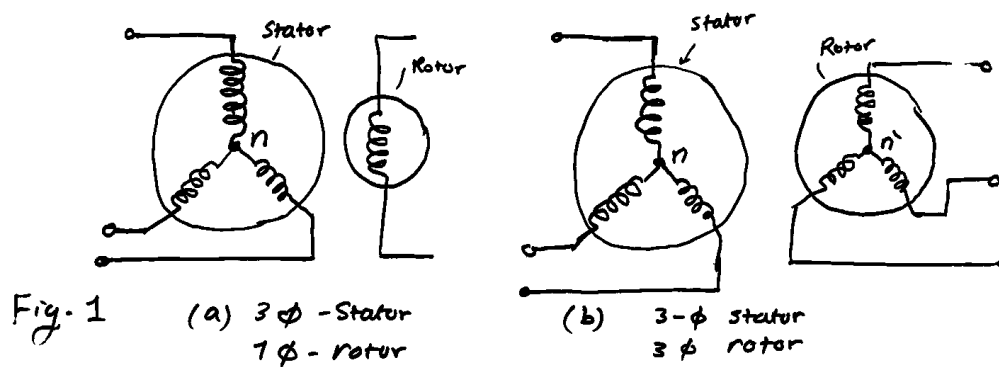
AC SERVO COMPONENTS

Synchro.

Definition: A small instrument, up to 10 cm diameter, used for the transmission and reception of both electrical and position data.

The basic synchro consists of a symmetrical three-phase stator and single-phase or three-phase rotor, Fig. 1(a) & (b) respectively

The stator consists of three identical windings, physically separated by 120° and star connected as shown in Fig. 1



Stator and rotor can be excited in variety of ways. Typical forms of synchros are listed below with reference to the connections (a) and (b) (Fig. 1) and the method of excitation :

1. Transmitter (CX) ... Connection (a)
2. Control Transformer (CTR) ... Connection (a)
3. Receiver (TR) ... Connection (a)
4. Differential Transmitter (CDX) ... Connection (b)
5. Differential Receiver (TDR) ... Connection (b)

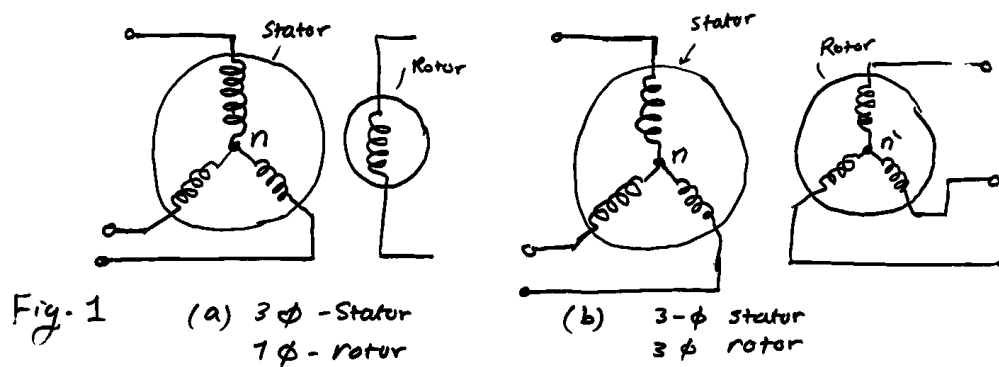
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4. Differential Transmitter (CDX) ... Connection (b)
5. Differential Receiver (TDR) ... Connection (b)

$$\begin{aligned}\phi_T &= \phi_R + \phi_B + \phi_Y = \phi_m \cos \theta \angle 0^\circ + \phi_m \cos(120 - \theta) \angle 120^\circ \\ &\quad + \phi_m \cos(240 - \theta) \angle 240^\circ \\ &= \underline{1.5 \phi_m \angle \theta}.\end{aligned}$$

If the receiver rotor is identical to that of the synchro transmitter; and excited by a reference voltage V_R but located at angle β with a reference axis (usually phase a or R axis), while the stator has a flux at angle θ , then a torque angle δ , equal to $\theta - \beta$ will exist between the stator and rotor fluxes (see Fig.4).

$$E_R = E_a = E \cos \theta$$

$$E_B = E_c = E \cos(240 - \theta)$$

$$E_Y = E_b = E \cos(120 - \theta)$$

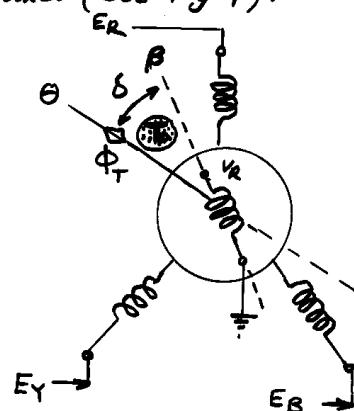


Fig.4

Once the torque is produced, the rotor turns until the fluxes are aligned. Thus, the electrical synchro data input to the stator receiver stator produces a mechanical shaft-angle Output. Generally, the receiver will respond to displacement of 1° .

Terminal Voltage Measurement

The tie point of the star-connected winding (point(n) in fig.1) is seldom accessible and, therefore, test measurements must be made between the terminals of the stator. These will be:

$$\begin{aligned}
 E_{ab} &= E_a - E_b = E \cos \theta - E \cos (120 - \theta) \\
 &= E \cos \theta - E \left(-\frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta \right) \\
 &= \sqrt{3} E \left(\frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta \right) \\
 &= \sqrt{3} E \sin (120 + \theta) .
 \end{aligned}$$

Similarly

$$E_{bc} = \sqrt{3} E \sin \theta$$

$$E_{ca} = \sqrt{3} E \sin (240 + \theta) .$$

(H.W.
for students)

Example:- The measured voltages between stator terminals are, $E_{ab} = 10.26$, $E_{bc} = 19.28$ and $E_{ca} = -29.54$. Find θ and E .

Solution

$$\begin{aligned}
 E_{ab} &= 10.26 = \sqrt{3} E \sin (120 + \theta) \\
 &= \sqrt{3} E \left[\frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta \right]
 \end{aligned}$$

$$\begin{aligned}
 E_{bc} &= 19.28 = \sqrt{3} E \sin \theta \\
 \frac{10.26}{19.28} &= \frac{\frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta}{\sin \theta} = \frac{\sqrt{3}}{2} \cot \theta - \frac{1}{2}
 \end{aligned}$$

$$\therefore \theta = \cot^{-1} \frac{2}{\sqrt{3}} \left(\frac{10.26}{19.28} + 0.5 \right) = 40^\circ$$

$$E = \frac{E_{bc}}{\sqrt{3} \sin \theta} = \frac{19.28}{\sqrt{3} \sin 40} = 17.3 \text{ Volts.}$$

H.W: Repeat the above example if $E_{ab} = -30V$,
 $E_{ab} = 20V$ and $E_{bc} = 10V$.

$$\text{Note: } \cos \alpha + \cos \beta = 2 \sin \frac{\beta + \alpha}{2} \sin \frac{\beta - \alpha}{2}$$

(Ans: $E = 17.64V$ & $\theta = 160.9^\circ$.)

Typical synchro systems connections (Transmitter + Receiver)

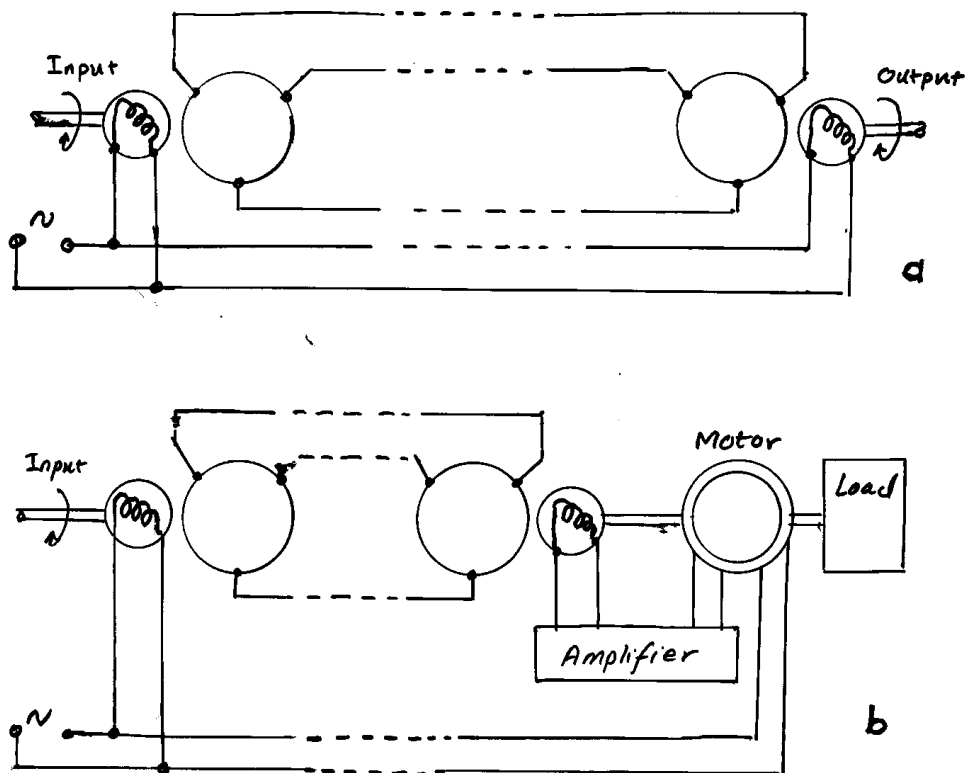
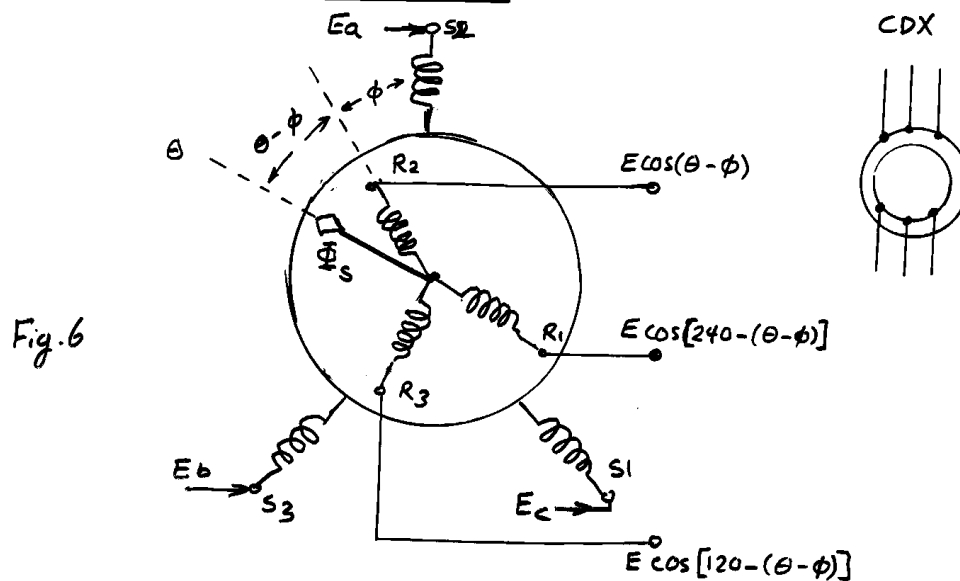


Fig. 5

Typical synchro systems are shown in Fig. . System (a) employs direct transmission of position data without feedback. The output synchro must be supplied with an operating torque sufficient to position the load and overcome friction. In (b) the loop is closed, operation is now on the angular error, and the load torque is supplied by the amplifier - motor combination.

Differential Transmitter :

1/cox



A rotor with three windings is shown in Fig. 6 above. The stator creates a flux Φ_s at the angle θ , and the rotor, whose reference axis is R_2 winding, is at angle ϕ with respect to the stator axis. Hence, the flux Φ_s is at angle $\theta - \phi$ with respect to the rotor, and the rotor voltages are synchro data for the difference angle. This is a synchro differential transmitter. It has two inputs :

- 1) electrical θ
 - 2) mechanical ϕ
- The output is electrical $(\theta - \phi)$.

Now, if we connect the CDX with a transmitter and receiver as shown in Fig. 7, a mechanical input to the transmitter θ is converted ^{to electrical} and applied to the stator of the differential transmitter; with a rotor

input ϕ , the electrical output of the differential transmitter is applied to the stator of the receiver to provide the mechanical output $\theta - \phi$.

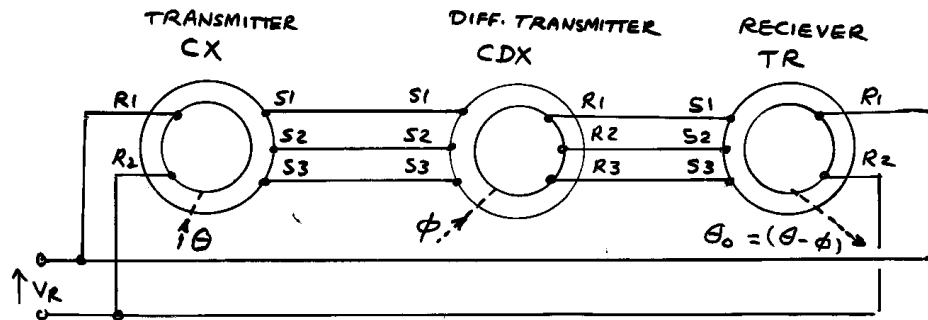


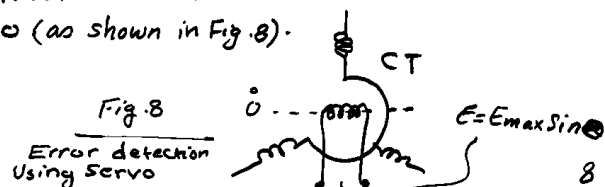
Fig.7 Data Transmission with CDX .

Applications :

synchros are extensively used in servomechanism and other applications. Two major applications are:

1. Torque Transmission .
2. Error Detection .

- Torque Transmission is achieved by using CX + TR. CX is work as a Master and TR is as slave. Any change in the position angle θ of the CX will let the rotor of the TR to follow this position. Therefore, torque transmission over a long distance is achieved without the use of a rigid mechanical connection (see Fig.3).
- Error detection is achieved by using CX as a master and CT (control transformer) as a slave. The control transformer CT is like the synchro receiver in construction but the rotor is held at its electrical zero (as shown in Fig.8).



Differential Receiver (TDR)

In construction, the synchro differential receiver is the same as the differential transmitter; however, it accepts two electrical input angles and provides the difference angle as a mechanical output.

In Fig. 10 below, two transmitters provide inputs to the differential receiver, one transmitter provides angle Θ to the stator and the other provides ϕ to the rotor (of the TDR). The reaction of the receiver is illustrated in Fig. 11. The torque angle between Φ_s and Φ_r (stator & rotor fluxes respectively) causes the rotor to turn until the fluxes are coincident. With both fluxes are at Θ , but the rotor axis is at $-\phi$ with respect to Φ_r , the rotor actual position then will be at the angle $\Theta - \phi$ relative to the stator axis (Fig. 11 (ii))

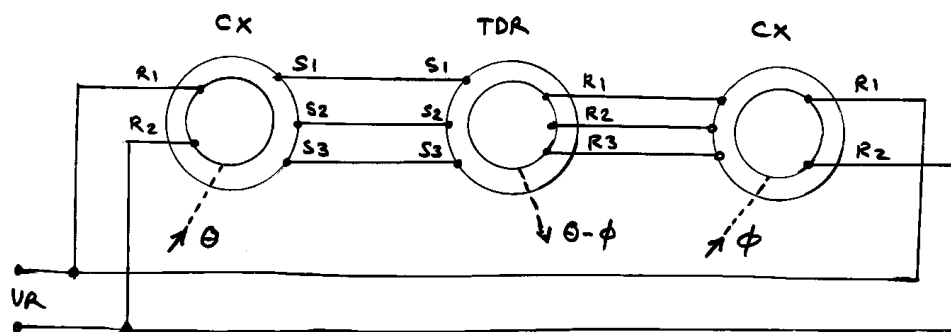


Fig. 10 : Data Transmission using TDR .

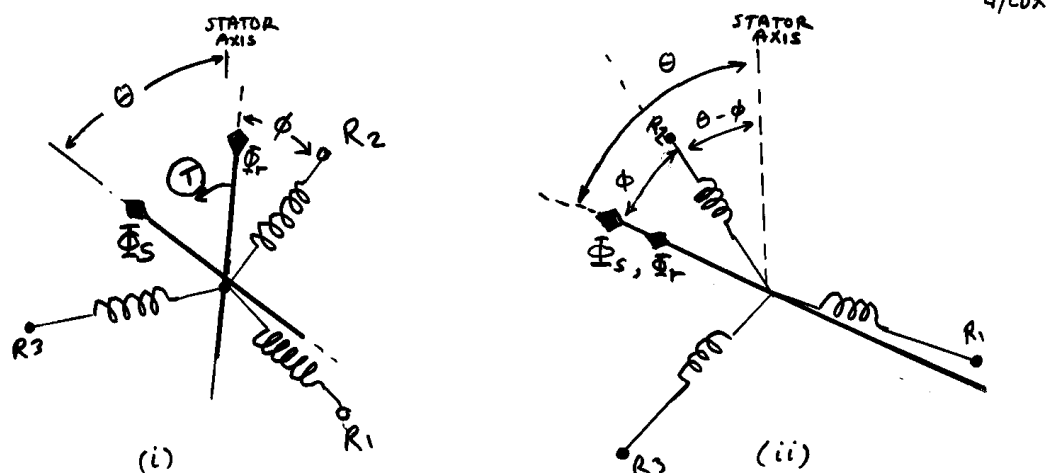
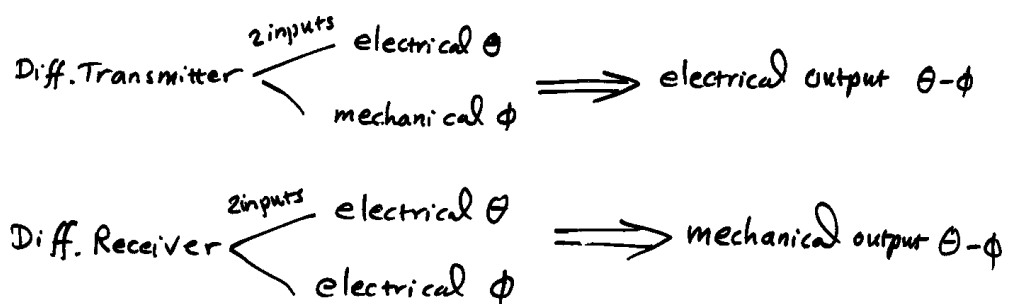


Fig.11: Rotor torque and response

The synchro transmitter, receiver, differential transmitter and differential receiver all have outputs - mechanical or electrical - that, remains fixed as long as there is no change in the output commands. They are data-transmission devices whose output can be introduced as a command to a feedback system in the same sense the input potentiometer provide a command to the dc positioning system.

Summary:



Small Electrical Machines

Tutorial Sheet No.2

Dr. M.T.Lazim

Question 1

In the data transmission system with CDX differential transmitter shown in Fig. 1, the reference voltage V_R for both the transmitter CX and the receiver TR is 20 volts r.m.s. If the input angle to the transmitter is 30° and the differential transmitter angle is 45° , find (i) the voltages V_{12} , V_{23} , and V_{31} across the receiver terminals. ii) the output angle)

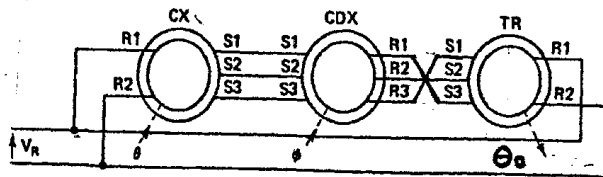


Fig.1

Question 2

The discussion of the synchro receiver showed that the electrical synchro data input created a flux of 1.5Φ on the axis θ . However, the development was based upon current flow resulting from the voltages induced in the transmitter coils only. Assuming identical electrical characteristics in both the transmitter and receiver, the current in each receiver winding is $(E_x - E_R)/2Z$, in which E_x and E_R are the rotor-induced voltages in corresponding phases of the transmitter and the receiver, respectively, and Z is the impedance of each winding. The resulting stator flux in the receiver is $\Phi_s/\phi = 1.5 \Phi/\theta - 1.5 \beta$. In the following table, ρ is the ratio $\Phi_s/1.5\Phi$ and $\Delta\theta$ is the angle by which angle ϕ leads angle θ ; thus, for an angular difference $(\theta - \beta)$, $\Phi_s/\phi = 1.5 \rho \Phi/\theta + \Delta\theta$.

$\theta - \beta$	ρ	$\Delta\theta$
0	0	—
30	0.518	75
60	1.0	60
90	1.414	45
120	1.732	30
150	1.932	15
180	2.0	0

- Verify that the torque on the rotor is in a direction to drive the receiver to angle θ .
- The torque acting on the rotor is $T_r = k \Phi_s \Phi_{\text{rotor}} \sin(\Phi - \theta)$. Show that $T_r = k_T \rho \sin[(\theta - \beta) + \Delta\theta]$, in which $k_T = 1.5 k \Phi \Phi_{\text{rotor}}$.
- Compute and plot the variable $\rho \sin[(\theta - \beta) + \Delta\theta]$ for the tabulated data. Discuss the plotted results.

Question 3 : Find θ_o of the systems shown in fig.2(a) and (b) .

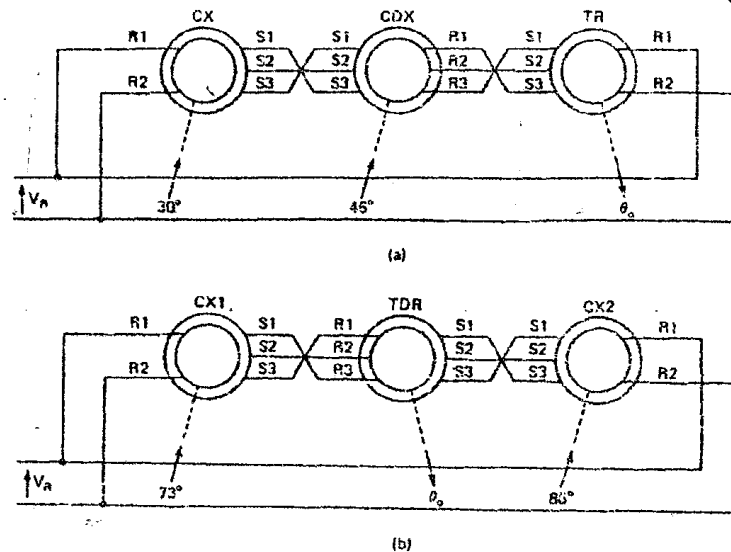


Fig.2

Question 4

For a synchro transmitter, the r.m.s. voltage induced in winding S_b is 50 V at the electrical zero position. When the rotor is at 30 degrees from electrical zero position in counterclockwise direction,

- Determine the r.m.s. voltages in the windings.
- Determine the r.m.s. voltages between lines.

(Ans.: (a)-43.3V, 43.3V, 0 ; (b)-86.6V, 43.3V, 43.3V)

Question 5:

A synchro transmitter (CX) and a synchro receiver (TR) are connected as master and slave to form data transmission system. The maximum r.m.s. voltage induced in the TR is 10 V. The rotor of the TR is held at the electrical zero position. Determine the voltage induced in the rotor of the TR in response to a displacement of the rotor of the CX from its electrical zero position by 20 degrees.

$\ominus$
 mechanical $\rightarrow$ electrical (θ) CX.
 electrical (θ) $\rightarrow$ mechanical (θ) TR. } one i/p } one o/p } Analog mic speaker

$\ominus$ Φ
 electrical + mechan $\rightarrow$ electrical ($\theta-\phi$) CDX } 2 i/p
 $\ominus$ Φ
 electrical + electrical $\rightarrow$ mechanical ($\theta-\phi$) TDR } 1 o/p.

Synchro	I/P	O/P
CX	Mechanical $\ominus$	Electrical $\ominus$
TR	Electrical $\ominus$	Mechanical $\ominus$
CDX	$\ominus$ Φ $E + M$	$E (\theta - \phi)$
TDR	$\ominus$ Φ $E + E$	$M (\theta - \phi)$

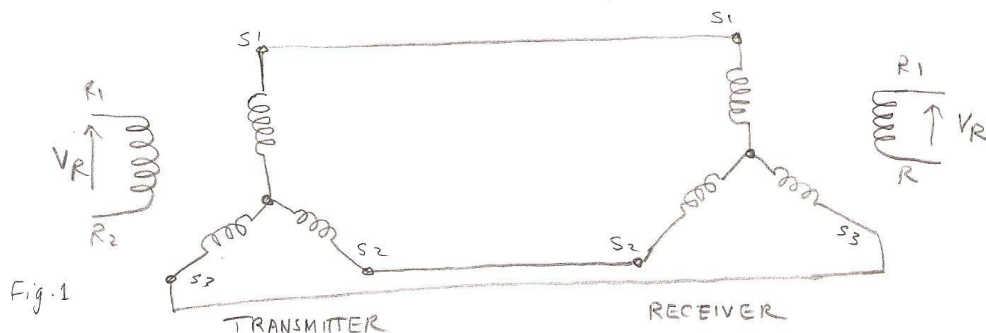
Homework problem

Q(a) Draw the connection of Synchros as Data Transmission Unit (DTU), explain the principle of operation and give necessary equations.

(b) for the DTU if $V_{12} = -30V$, $V_{31} = 20V$ and $V_{23} = 10V$
Find E and θ .

Solution

(a) The synchro is a motor-like component used for data transmission. It consists of rotor (single-phase winding) and a stator (3-phase winding) and it operates in couple arrangement (transmitter + receiver) as shown in figure below (Fig.1).



The single unit is represented as shown in Fig. 2. It is clear that due to the orientation of the rotor (given as input to the rotor), the voltages induced in the three stator windings are:

$$V_{S1} = E \cos \theta$$

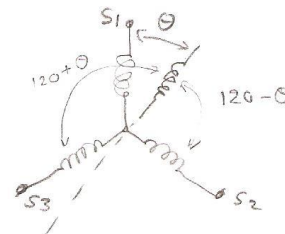
$$V_{S2} = E \cos(120 - \theta)$$

$$V_{S3} = E \cos(120 + \theta)$$

where $E = E_m \cos \omega t$

Hence, the voltage in the stator will carry information about the input angle θ .

Then if this voltage is applied to the stator of the receiver will get currents of :



$$I_1 = \frac{E \cos \theta - V_n}{|Z|}, \quad I_2 = \frac{E \cos(120 - \theta) - V_n}{|Z|}, \quad I_3 = \frac{E \cos(120 + \theta) - V_n}{|Z|}$$

$$\text{or } I_1 = I \cos \theta - \frac{V_n}{Z}, \quad I_2 = I \cos(120 - \theta) - \frac{V_n}{Z}, \quad I_3 = I \cos(120 + \theta) - \frac{V_n}{Z}$$

$$\text{where } I = \frac{E}{|Z|}$$

$$\text{Since } I_1 + I_2 + I_3 = 0 \quad (\text{By KCL}).$$

$$\text{Hence } \frac{V_n}{Z} = 0$$

$$\text{or } I_1 = I \cos \theta, \quad I_2 = I \cos(120 - \theta), \quad I_3 = I \cos(120 + \theta)$$

These currents will produce corresponding fluxes, Φ_1, Φ_2 and Φ_3 as follows:

$$\Phi_1 \propto I \cos \theta, \quad \Phi_2 \propto I \cos(120 - \theta) \quad \& \quad \Phi_3 \propto I \cos(120 + \theta)$$

$$\begin{aligned} \text{Total flux } \Phi_s &= \Phi_1 + \Phi_2 + \Phi_3 \\ &= K I [\cos \theta + \cos(120 - \theta) + \cos(120 + \theta)] \\ &= \Phi_{\max} [\cos \theta + \cos(120 - \theta) + \cos(120 + \theta)] \end{aligned}$$

$$\text{where } \Phi_{\max} = K I$$

$$\therefore \Phi_s = 1.5 \Phi_{\max} \angle \theta$$

But if we activate this rotor by the same voltage V_R given to the rotor of the transmitter and suppose that R_1 is misaligned by β from S_1 , then the resultant flux is

$$\Phi_s = 1.5 \Phi_{\max} \angle \theta - \beta$$

So this angle (called torque angle) will produce a torque which rotates the receiver rotor until we get $\theta - \beta = 0$ or $\beta = \theta$ here the value of θ is transmitted to the receiver very efficiently.

(b) If $V_{12} = -30V$, $V_{31} = 20V$ & $V_{23} = 10V$.

We have in general

$$\cos \alpha - \cos \beta = 2 \sin \frac{\alpha + \beta}{2} \sin \frac{\beta - \alpha}{2}$$

$$\begin{aligned} V_{12} &= V_1 - V_2 = E [\cos \theta - \cos (\theta - 120^\circ)] \\ &= E \left[2 \sin \frac{(\theta + \theta - 120^\circ)}{2} \sin \frac{\theta - \theta - 120^\circ}{2} \right] \\ &= E \left[2 \sin \frac{(2\theta - 120^\circ)}{2} \sin (-60^\circ) \right] \\ &= E \left[-2 \frac{\sqrt{3}}{2} \sin (\theta - 60^\circ) \right] \\ &= \sqrt{3} E \sin (-(\theta - 60^\circ)) = \sqrt{3} E \sin (60^\circ - \theta) \\ &= \sqrt{3} E (\sin 60^\circ \cos \theta - \sin \theta \cos 60^\circ) \\ &= \sqrt{3} E \left(\frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta \right) = -30 \quad \dots (1) \end{aligned}$$

since
 $\cos(120^\circ - \theta) = \cos(\theta - 120^\circ)$

Similarly:

$$\begin{aligned} V_{31} &= V_3 - V_1 = E [\cos (120^\circ + \theta) - \cos \theta] \\ &= E \left[2 \sin \frac{-120^\circ}{2} \sin \frac{2\theta + 120^\circ}{2} \right] \\ &= E \left[-2 \frac{\sqrt{3}}{2} \sin (\theta + 60^\circ) \right] \\ &= E [-\sqrt{3} \{ \sin \theta \cos 60^\circ + \sin 60^\circ \cos \theta \}] \\ &= E \left[-\sqrt{3} \times \frac{1}{2} \sin \theta - \frac{\sqrt{3} \times \sqrt{3}}{2} \cos \theta \right] = 20 \quad \dots (2) \end{aligned}$$

$$\begin{aligned} V_{23} &= V_2 - V_3 = E [\cos (120^\circ - \theta) - \cos (120^\circ + \theta)] \\ &= E \left[2 \sin \frac{(120^\circ - \theta + 120^\circ + \theta)}{2} \sin \frac{(120^\circ + \theta - 120^\circ - \theta)}{2} \right] \\ &= E [2 \sin 120^\circ \sin \theta] \\ &= E \left[2 \frac{\sqrt{3}}{2} \sin \theta \right] = \sqrt{3} E \sin \theta = 10 \quad \dots (3) \end{aligned}$$

Dividing Eq. (1) by (2):

$$\frac{V_{12}}{V_{23}} = \frac{-30}{10} = \frac{\sqrt{3} E \left(\frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta \right)}{\sqrt{3} E \sin \theta} = \frac{\sqrt{3}}{2} \frac{\cos \theta}{\sin \theta} - \frac{1}{2} \frac{\sin \theta}{\sin \theta}$$

$$-3 = \frac{\sqrt{3}}{2} \cot \theta - \frac{1}{2}$$

$$\therefore \cot \theta = -3 + \frac{1}{2} \bigg/ \frac{\sqrt{3}}{2} = \frac{-2.5}{\frac{\sqrt{3}}{2}} = \frac{-2.5}{0.866} = -2.886$$

(b) If $V_{12} = -30V$, $V_{31} = 20V$ & $V_{23} = 10V$.

we have in general

$$\cos \alpha - \cos \beta = 2 \sin \frac{\alpha + \beta}{2} \sin \frac{\beta - \alpha}{2}$$

$$V_{12} = V_1 - V_2 = E [\cos \theta - \cos (\theta - 120^\circ)]$$

$$= E \left[2 \sin \frac{(\theta + \theta - 120^\circ)}{2} \sin \frac{\theta - \theta - 120^\circ}{2} \right]$$

$$= E \left[2 \sin \frac{(2\theta - 120^\circ)}{2} \sin (-60^\circ) \right]$$

$$= E \left[-2 \frac{\sqrt{3}}{2} \sin (\theta - 60^\circ) \right]$$

$$= \sqrt{3} E \sin (-(\theta - 60^\circ)) = \sqrt{3} E \sin (60^\circ - \theta)$$

$$= \sqrt{3} E (\sin 60^\circ \cos \theta - \sin \theta \cos 60^\circ)$$

$$= \sqrt{3} E \left(\frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta \right) = -30 \quad \dots (1)$$

Similarly:

$$V_{31} = V_3 - V_1 = E [\cos (120^\circ + \theta) - \cos \theta]$$

$$= E \left[2 \sin \frac{-120^\circ}{2} \sin \frac{2\theta + 120^\circ}{2} \right]$$

$$= E \left[-2 \frac{\sqrt{3}}{2} \sin (\theta + 60^\circ) \right]$$

$$= E [-\sqrt{3} \{ \sin \theta \cos 60^\circ + \sin 60^\circ \cos \theta \}]$$

$$= E \left[-\sqrt{3} \times \frac{1}{2} \sin \theta - \frac{\sqrt{3} \times \sqrt{3}}{2} \cos \theta \right] = 20 \quad \dots (2)$$

$$V_{23} = V_2 - V_3 = E [\cos (120^\circ - \theta) - \cos (120^\circ + \theta)]$$

$$= E \left[2 \sin \frac{(120^\circ + \theta + 120^\circ + \theta)}{2} \sin \frac{(120^\circ + \theta - 120^\circ - \theta)}{2} \right]$$

$$= E [2 \sin 120^\circ \sin \theta]$$

$$= E \left[2 \frac{\sqrt{3}}{2} \sin \theta \right] = \sqrt{3} E \sin \theta = 10 \quad \dots (3)$$

Dividing eq. (1) by (2):

$$\frac{V_{12}}{V_{23}} = \frac{-30}{10} = \frac{\sqrt{3} E \left(\frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta \right)}{\sqrt{3} E \sin \theta} = \frac{\sqrt{3}}{2} \frac{\cos \theta}{\sin \theta} - \frac{1}{2} \frac{\sin \theta}{\sin \theta}$$

$$-3 = \frac{\sqrt{3}}{2} \cot \theta - \frac{1}{2}$$

$$\therefore \cot \theta = -3 + \frac{1}{2} \bigg/ \frac{\sqrt{3}}{2} = \frac{-2.5}{\frac{\sqrt{3}}{2}} = \frac{-2.5}{0.866} = -2.886$$

$$\text{But } \tan \theta = \frac{1}{\cot \theta} = \frac{1}{-2.886} = -0.3464$$

$$\therefore \theta = \boxed{-19^\circ} \quad \text{or} \quad \boxed{160.9^\circ}$$

$$E = \frac{10}{\sqrt{3} \sin 160.9} = \frac{10}{\sqrt{3} \times 0.32} = \boxed{17.64} \text{ Volts}$$

$$\text{Hence } \theta = \boxed{160.9^\circ}$$

Tutorial Sheet No-2

Question-1

Assuming the coupling factor between the rotor and stator is ($k=1$) for simplification, we get:

At CX

$$V_{S1} = 20 \cos \theta$$

$$V_{S2} = 20 \cos (120 - \theta)$$

$$V_{S3} = 20 \cos (120 + \theta)$$

Given to CDX

$$V_{R1} = 20 \cos (\theta - \phi)$$

$$V_{R2} = 20 \cos [120 - (\theta - \phi)]$$

$$V_{R3} = 20 \cos [120 + (\theta - \phi)]$$

Given to TR yields:

$$V_{R3} \Rightarrow V_{S1}$$

$$\therefore V_{S1} = 20 \cos [120 + (\theta - \phi)]$$

$$V_{R2} \Rightarrow V_{S2}$$

$$\therefore V_{S2} = 20 \cos [120 - (\theta - \phi)]$$

$$V_{R1} \Rightarrow V_{S3}$$

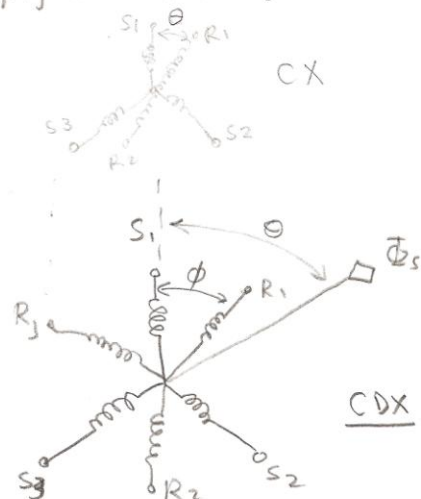
$$\therefore V_{S3} = 20 \cos (\theta - \phi)$$

To get V_{12} , V_{23} & V_{31} , we have the identity

$$\cos A - \cos B = 2 \sin \frac{A+B}{2} \sin \frac{B-A}{2}$$

So

$$\begin{aligned} V_{12} &= V_{S1} - V_{S2} = 20 [\cos (120 + (\theta - \phi)) - 20 \cos [120 - (\theta - \phi)]] \\ &= 20 \times 2 \sin \frac{240}{2} \sin \frac{-(\theta + \phi) - (\theta - \phi)}{2} \\ &= 40 \sin 120 \sin [-(\theta - \phi)] \end{aligned}$$



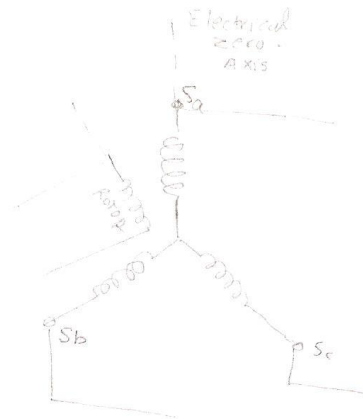
In this graph
It is assumed that
 $\theta > \phi$.

a) For $\theta = 30^\circ$

$$E_{sa} = E \cos \theta$$

$$E_{sb} = E \cos(120 - \theta)$$

$$E_{sc} = E \cos(240 - \theta)$$



$$E_{sa} = 50 \cos 30 = 50 \times \frac{\sqrt{3}}{2} = 43.3 \text{ V}$$

$$E_{sb} = 50 \cos(120 - 30) = 50 \cos 90 = 0 \text{ V}$$

$$E_{sc} = 50 \cos(240 - 30) = 50 \cos 210 \\ = 50 \cos(180 + 30) = -50 \cos 30 = -50 \times \frac{\sqrt{3}}{2} = -43.3 \text{ V}$$

b) line to line voltages:

$$E_{ab} = E_a - E_b = 43.3 - 0 = 43.3 \text{ V}$$

$$E_{bc} = E_b - E_c = 0 - (-43.3) = 43.3 \text{ V}$$

$$E_{ca} = E_c - E_a = -43.3 - 43.3 = -86.6 \text{ V}$$



$$\cos x = 1$$

$$\sin x(t) \quad \left| \begin{array}{l} \text{Max} \\ \cos x \\ \text{Max} \end{array} \right| \quad \left| \begin{array}{l} \text{Max} \\ \cos x \\ \text{Max} \end{array} \right| \quad \left| \begin{array}{l} \text{Max} \\ \cos x \\ \text{Max} \end{array} \right|$$

$$\tan x(t) \quad \left| \begin{array}{l} \cos x(t) \end{array} \right|$$

$$\text{or: } E_{ab} = E_a - E_b = E \cos \theta - E \cos(120 - \theta)$$

$$= E (\cos \theta - \cos 120 \cos \theta - \sin 120 \sin \theta)$$

$$= E [\cos \theta - (-) \sin 30 \cos \theta - \cos 30 \sin \theta]$$

$$= E [\cos \theta + \frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta]$$

$$= E [\frac{3}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta]$$

$$= \sqrt{3} E [\frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta]$$

$$= \sqrt{3} E [\sin 120 \cos \theta + \cos 120 \sin \theta]$$

$$= \sqrt{3} E [\sin(120 + \theta)]$$

$$= \sqrt{3} \times 50 \sin(120 + 30) = \sqrt{3} \times 50 \sin 150$$

$$= \sqrt{3} \times 50 \sin(180 - 30) = \sqrt{3} \times 50 \sin 30$$

$$= \sqrt{3} \times 50 \times \frac{1}{2} = 0.866 \times 50 = \underline{\underline{43.3 \text{ V}}}$$

$$\cos 120$$

$$= \cos(90 + 30)$$

$$= -\sin 30$$

$$= -\frac{1}{2}$$

$$\sin 120$$

$$= \sin(90 + 30)$$

$$= \cos 30$$

$$= \frac{\sqrt{3}}{2}$$

$$\cos 240$$

$$= \cos(180 + 60)$$

$$= -\cos 60$$

$$= -\frac{1}{2}$$

$$\sin 240$$

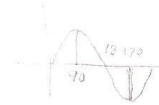
$$= \sin(180 + 60)$$

$$= -\sin 60$$

$$= -\frac{\sqrt{3}}{2}$$

$$\begin{aligned}
 E_{bc} &= E_b - E_c = E \cos(120 - \theta) - E \cos(240 - \theta) \\
 &= E [\cos(120 - \theta) - \cos(240 - \theta)] \\
 &= E [\cos 120 \cos \theta + \sin 120 \sin \theta - \cos 240 \cos \theta - \sin 240 \sin \theta] \\
 &= E \left[-\frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta - \left(-\frac{1}{2}\right) \cos \theta - \left(-\frac{\sqrt{3}}{2}\right) \sin \theta \right] \\
 &= E \left[\frac{\sqrt{3}}{2} \sin \theta + \frac{\sqrt{3}}{2} \sin \theta \right] = \sqrt{3} E \sin \theta \\
 &= \sqrt{3} \times 50 \times \sin 30 = \frac{\sqrt{3}}{2} \times 50 = \underline{43.3 \text{ V}}
 \end{aligned}$$

$$\begin{aligned}
 E_{ca} &= E_c - E_a = E \cos(240 - \theta) - E \cos \theta \\
 &= E [\cos 240 \cos \theta + \sin 240 \sin \theta - \cos \theta] \\
 &= E \left[-\frac{1}{2} \cos \theta + \left(-\frac{\sqrt{3}}{2}\right) \sin \theta - \cos \theta \right] \\
 &= E \left[-\frac{3}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta \right] \\
 &= -\sqrt{3} E \left[-\frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta \right] \\
 &= \sqrt{3} E [\sin 240 \cos \theta + \cos 240 \sin \theta] \\
 &= \sqrt{3} E [\sin(240 + \theta)] - \quad (-) \\
 &= \sqrt{3} \times 50 \sin(240 + 30) = \sqrt{3} \times 50 \times \sin 270 \\
 &= -\sqrt{3} \times 50 = -86.6 \text{ V}
 \end{aligned}$$



Summary:

phase voltages for CX:

$$E_a = E \cos \theta$$

$$E_b = E \cos(120 - \theta)$$

$$E_c = E \cos(240 - \theta)$$

line-line voltages:

$$E_{ab} = \sqrt{3} E \sin(120 + \theta)$$

$$E_{bc} = \sqrt{3} E \sin \theta$$

$$E_{ca} = \sqrt{3} E \sin(240 + \theta)$$

$$\begin{aligned}
 V_{23} &= V_{s2} - V_{s3} = 20 \cos[120 - (\theta - \phi)] - 20 \cos(\theta - \phi) \\
 &= 20 \times 2 \sin \frac{120}{2} \sin \frac{(\theta - \phi) + (\theta - \phi) - 120}{2} \\
 &= 40 \sin 60 \sin [(\theta - \phi) - 60]
 \end{aligned}$$

$$\begin{aligned}
 V_{31} &= V_{s3} - V_{s1} = 20 \cos(\theta - \phi) - 20 \cos[120 + (\theta - \phi)] \\
 &= 20 \times 2 \sin \frac{120 + (\theta - \phi) + (\theta - \phi)}{2} \sin \frac{120}{2} \\
 &= 40 \sin [(\theta - \phi) + 60] \sin 60
 \end{aligned}$$

For $\theta = 30^\circ$, $\phi = 45^\circ$ we get :

$$\begin{aligned}
 V_{12} &= 40 \sin 120 \sin [-(30 - 45)] = 8.96 \text{ Volts} \\
 V_{23} &= 40 \sin 60 \sin [(30 - 45) - 60] = -33.46 \text{ Volts} \\
 V_{31} &= 40 \sin 60 \sin [(30 - 45) + 60] = 24.49 \text{ Volts}
 \end{aligned}
 \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \text{Answer} \\ (b) \end{array}$$

Now to find the output angle Let us plot the resultant stator flux as shown below :

Taking S_3 as reference :

Input angle $= \theta - \phi$

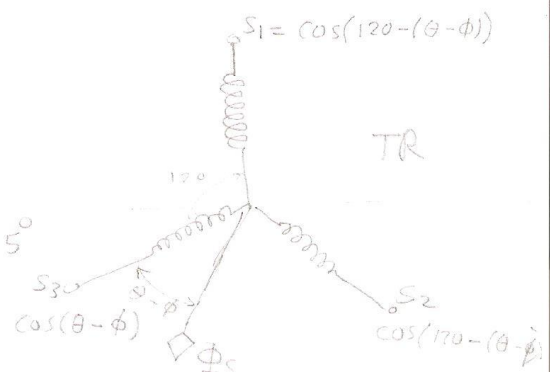
$\theta = 30^\circ$, $\phi = 45^\circ$ yields

Output angle $\theta_o = 30 - 45 = -15^\circ$

Taking S_1 as reference

$$\theta_o = 120 + 15 = 135^\circ$$

$$\theta_o = 120 + (-15) = 105^\circ$$



Exam. CUV direction Sa

Q2.

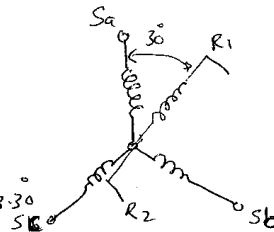
①

$$S_a = 50V$$

$$V_{S1} = E \cos \theta = 50 \cos 30 = 43.3^\circ$$

$$V_{S2} = E \cos(120 - \theta) = 50 \cos 90 = 0$$

$$V_{S3} = E \cos(120 + \theta) = 50 \cos 150 = -43.3^\circ$$



②

$$V_{12} = V_1 - V_2 = E \cos \theta - E \cos(120 - \theta)$$

$$= 50 [\cos \theta - \cos(120 - \theta)]$$

$$= 50 \times 2 \left[\sin \frac{(\theta + 120 - \theta)}{2} \sin \frac{(120 - \theta - \theta)}{2} \right]$$

$$= 100 [\sin 60 \sin(60 - \theta)]$$

$$= 100 \left[\frac{\sqrt{3}}{2} \sin(60 - 30) \right] = \frac{100\sqrt{3}}{2} \times \frac{1}{2} = 25\sqrt{3} = 43.30^\circ$$

$$V_{23} = V_2 - V_3 = E \cos(120 - \theta) - E \cos(120 + \theta)$$

$$= 50 [\cos(120 - \theta) - \cos(120 + \theta)]$$

$$= 2 \times 50 \left[\sin \frac{(120 - \theta + 120 + \theta)}{2} \sin \frac{(120 - \theta - 120 + \theta)}{2} \right] \quad (\beta - \alpha) \text{ or } (\alpha - \beta) \text{ check plus}$$

$$= 100 [\sin 60 \sin \theta] = 100 \frac{\sqrt{3}}{2} \times \frac{1}{2} = 43.3^\circ \text{ V (or } -43.3^\circ)$$

$$V_{13} = V_1 - V_3 = E \cos \theta - E \cos(120 + \theta)$$

$$= 50 [\cos \theta - \cos(120 + \theta)]$$

$$= 50 \times 2 \left[\sin \frac{(\theta + 120 + \theta)}{2} \sin \frac{(120 + \theta - \theta)}{2} \right]$$

$$= 86.6 \text{ V}$$

- Q: For a synchro transmitter, the r.m.s voltage induced in winding S_a is 50 V at the electrical zero position. When the rotor is at 30 degrees from electrical zero position in clock wise direction.
- Determine the r.m.s. voltages in the windings
 - Determine the r.m.s voltages between lines.